Volume 5, Issue 5
October 2016

ITEE Journal
Information Technology & Electrical Engineering

ISSN: - 2306-708X



# Mathematical Modelling, and Controller Design for Boiler Plant With Long Delay Time

[1]Dinesh Singh Rana and [2]Diksha Soni
[1,2]Department of Instrumentation, Kuruskhetra University, Kurukshetra, Haryana, INDIA
E-mail: [1]dineshsinghrana24@rediffmail.com

## ABSTRACT

This paper focuses on the modelling, analysis and designing of controllers for boiler plant with long delay time. In continuous time delayed system, both of Pad'e approximation and Smith predictor techniques are used to model the delay. In each case, the controllers were designed using two important strategies which include PID and pole placement methods based on Algebraic Riccati Equation (ARE). In Pad'e method, the delay is modelled as a rational transfer function. In Smith predictor, the delay is shifted outside the feedback loop and the system may be considered as a delay free system with limited constraints. The simulation results indicates that the ARE approach gives better performance as compare to PID based methods for optimal controller performance.

**Keywords:** *Boiler plant, Dead time, Smith Predictor (SP), PID, Algebraic Riccati Equation (ARE), Time Delay.*

## 1. INTRODUCTION

The problem Dead Time in control systems is an everlasting problem which is of primary importance in process control as well. Any delay in measuring, in controller action, or in actuator operation is termed as dead time. It reduces the stability of system and reduces the achievable response time of system. The presence of dead times in the control loops have two main consequences. It greatly complicates the analysis and the design of feedback controllers. It is also difficult to attain satisfactory control performance. The time delays increase the phase lag which leads instability of the control system at relatively lower controller gain. As a result, delay time put constrains on the performance of the control process. Boiler drum is commonly used in industries in almost all process and power plants to generate steam for the main purpose of electricity generation via steam turbines [1]. The dead time is the problem that also affects the performance of boiler drum. The objective of the drum level control system is to maintain the water-steam interface at the specified level and provide a continuous mass balance by replacing every pound of steam and water removed with a pound of feed water [2]. As , low level affects the recirculation of water to the boiler tubes and reduces the water to the boiler tubes, which overheats and can cause damage to the boiler tubes. High level reduces the surface area, and can lead to water and dissolved solids entering the steam distribution system. The level control of a boiler is non- linear, time varying and time delay system. Time delayis defined as the required time between applying change in the input and notices its effect on the system output. Generally, dead times are caused by the time needed to transport energy or information [3]. In order to obtain the desired performance it is necessary to compensate the time delay or dead time. Time

delay is compensated by using two techniques PID and Algebraic Riccati Equation (ARE) and its code is implemented to control the delay in boiler system using MATLAB/ SIMULINK approach.

## 2. MATHEMATICAL MODELLING

Systems with delay have infinite dimensions which make it impossible to express the system in state space. So there is a need to model the delay which has been done by two approaches which includes Pad'e approximation and Smith predictor method. Figure 1 shows the P&ID diagram of boiler system.

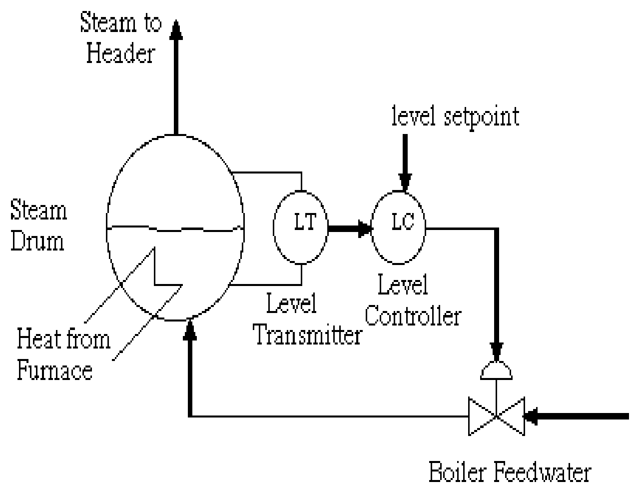



Figure 1:P & ID Diagram of Boiler System.

The transfer function for Boiler Plant without equation(1) and with dead time can be written as equation(2),

$$G(s)=\frac{0.31s+0.103}{s^2+0.25s+0.0083} \quad (1)$$

$$G(s)=\frac{0.31s+0.103}{s^2+0.25s+0.0083}e^{-sh} \quad (2)$$

## 2.1PAD'E APPROXIMATION METHOD

The Pad'e approximation approximates a pure time delay by a rational transfer function which simplifies the analysis and design of time-delay system [4]. The approximation enables the delay system to be treated as delay-free system. The Pad'e approximation for the term $e^{-sh}$ is given by equation (3)

$$e^{-sh}=Nr\,(sh)/Dr(sh) \quad (3)$$

*Where*

$$Nr(sh) \;=\sum_{k=0}^{r}\frac{(2r-k)!}{k!(r-k)!}(-sh)^k \quad (4)$$

$$Dr(sh) \;=\sum_{k=0}^{r}\frac{(2r-k)!}{k!(r-k)!}(-sh)^k \quad (5)$$

The equation (3) is written as

$$e^{-sh}=\frac{1-\frac{h}{2}s+\frac{h^2}{12}s^2}{1+\frac{h}{2}s+\frac{h^2}{12}s^2} \text{ (Second order)} \quad (6)$$

## 2.2 SMITH PREDICTOR METHOD

In Smith predictor, the delay is shifted outside the feedback loop and the system may be considered as a delay free system with certain constraints [5]. A feedback control system with a time delay is shown in Figure 2, Where C(s) is the controller; $G_0(s)e^{-sh}$ is the plant with a time delay h, where all zeros and poles of $G_0(s)$ are in the left half plane; d is the disturbance.

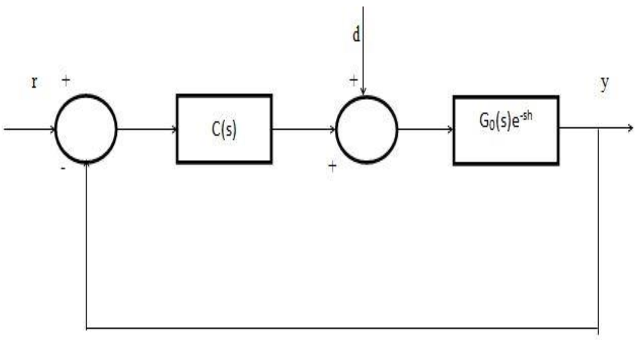



Figure 2:Feedback control system with a time delay.

In this case, the transfer function of the closed-loop system with the output y(s) and input r(s) can be formulated as (7)

$$Y\,(s)/r(s)=C(s)G_0(s)e^{-sh}/\left(1+C(s)\;G_0(s)e^{-sh}\right) \quad (7)$$

From above equation, it is very clear that the location of the closed-loop poles directly related to the time delay h. As result, the stability of the system can be affected by theamount the delay. The classical configuration of a system containing a Smith predictor is depicted in Figure 3.

Where

$$G(s)=G_0(s)e^{-sh}\ ,$$

$G_0'(s)$ and $G'(s)$ are nominal modes of $G_0(s)$ and $G(s)$ respectively.

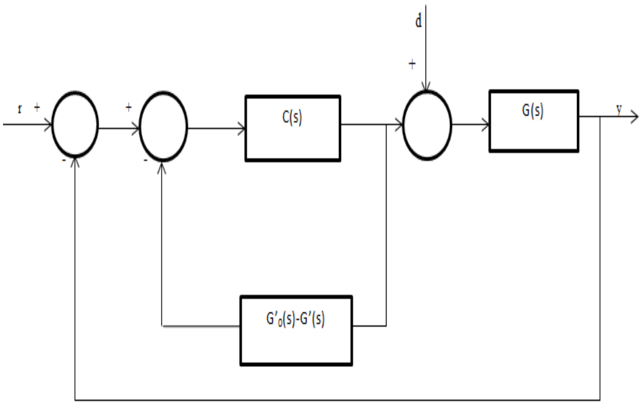



Figure 3:Smith Predictor.

If we assume the perfect model matching i.e. $G'(s)=G(s)$, the closed-loop transfer function becomes

$$Y(s)/R(s)=C(s)G(s)\div 1+C(s)\left[G_0'(s)-G'(s)\right]/1+ \quad (8)$$
$$C(s)G(s)\div 1+C(s)\left[G_0'(s)-G'(s)\right]$$

The equation (8) can be rearranged as

$$\frac{Y(s)}{R(s)}=\frac{C(s)G(s)}{1+C(s)G_0(s)}e^{-sh} \quad (9)$$

## 2.3 PID APPROXIMATION

Let the $e^{-sh}$ can be written as

$$e^{-sh}=\frac{1-0.5sh}{1+0.5sh} \quad (10)$$

Using pad'e approximation (for r =1), then$C_{eq}$ for real PID controller is given as

$$C_{pid}\;=\;\frac{k_c(1+sT_i)(1+sT_d)}{sT_i(1+\alpha\;s\;T_d)} \quad (11)$$

Where

$$T_i\;=\;T\left[\;\frac{1-\;(2T-h)(T-T_0)}{(2T+h)\;T^2}\;\right], \qquad K_c\;=\;\frac{T_i}{k_p(h+2T_0-T_i)}$$

,

and $\alpha\;=\;T_0\,/\;h+T_0+hT_i\div 2T_0-\;hT_0\div 2T$ .

## 2.4 POLE PLACEMENT METHOD (ALGEBRAIC RICCATI EQUATION (ARE)

The ARE is given by,

$$PA\;-\;PRP\;+\;Q\;=\;0 \quad (12)$$

Where, A, Q, R, ε $R^{n*n}$, where Q=Q′ and R=R′.

Volume 5, Issue 5
October 2016

ITEE Journal
Information Technology & Electrical Engineering

ISSN: - 2306-708X



The above ARE can also be written in the matrix form

$$\begin{bmatrix} -P & I \end{bmatrix}\begin{pmatrix} A & -R \\ -Q & -A' \end{pmatrix}\frac{I}{P} = 0 \quad (13)$$

The Riccati Equation is a matrix generalization of the standard quadratic equation. In the $1\times 1$ case, it becomes

$$ap + pa - prp + q = -rp^2 + 2ap + q = 0 \quad (14)$$

This scalar quadratic equation has two solutions p that are represented as

$$p = \frac{-2a \pm \sqrt{4a^2 + 4rq}}{-2r} = \frac{a \pm \sqrt{a^2 + rq}}{r} \quad (15)$$

Note that $-rp = \frac{a \pm \sqrt{a^2+rq}}{r} = \frac{\sqrt{a^2+rq}}{r}$ there are two possible values of a-rp, both symmetric about origin.

The eigenvalues of RP will correspond to poles of the closed-loop system. We are interested only in stable closed loop systems. There is only one of the systems which have $\frac{(2n)!}{n!n!}$ possible solutions that make A-RP stable by placing all its eigenvalues and hence the poles in the left-half plane [6]. In the 1x1 case above, $-rp = \frac{\pm\sqrt{a^2+rq}}{1}$, so only one solution of p can satisfy a-rp<0.

Every ARE ( $A'P + PA - PRP + Q = 0$ ) has an associated Hamiltonian matrix

$$H = \begin{pmatrix} A & -R \\ -Q & -A' \end{pmatrix} \varepsilon\ R^{2n*2n} \quad (16)$$

The notation P = Ric (H) to denotes the one solution to the ARE which makes *A*-RP stable.So we have included only those choices for H which has stabilizing solutions P satisfying following conditions

$$P = P' \quad (17)$$

$$A'P + PA - PRP + Q = 0 \quad (18)$$

$$R_e\left[\lambda_i\left(A - RP\right)\right] < 0 \quad (19)$$

Consider the linear time-invariant controllable system

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (20)$$

$$x(t_0) \cong x^0 \quad (21)$$

With linear control law of the form

$$U = -kx$$

(22)

(22)

The feedback gain matrix K may be selected to place the poles of the closed-loop system $\dot{x}(t) = (A - BK)x(t)$ at certain desired locations.

## 3. SIMULATION AND DESIGN OF CONTROLLER

The frequency response of transfer function is determined by plotting magnitude (M) and phase angle (Φ) against frequency. H.W.Bode suggested the graphs to be plotted (m) and (Φ) over a wide range of frequencies [7]. The main idea of frequency based design is to use the bode plot of the open loop transfer function to estimate the closed loop response. A Bode diagram consists of two graphs: one is a plot of the logarithm of the magnitude of sinusoidal transfer function; the other is a plot of the phase angle; both are plotted against the frequency on a logarithmic scale in the simulation. The standard representation of the logarithmic magnitude of G(jω) is $20\log_{10}G(j\omega)$, where the base of the logarithm is 10. The unit used in this representation of the magnitude is the decibel, usually abbreviated dB. The logarithmic representation, the curves are drawn on semi log paper, using the log scale for the frequency and the linear scale for either magnitude (dB) or phase angle (degrees). The main advantage of using bode diagram is that the multiplication of magnitudes can be converted into addition [8, 9]. Bode Plot is basically the rectangular plot. So we have use the bode plot in simulation for analysis of analysis of Phase and Magnitude Response of the Boiler System. Figure 4 shows the smith predictor step response indicating rise time (a), settling time (b) and peak amplitude (c). Phase and Magnitude Response of the Boiler System is shown in Figure 5. The Bode Plot shows that the gain margin is negative and phase margin is positive, so the system will be open loop stable.

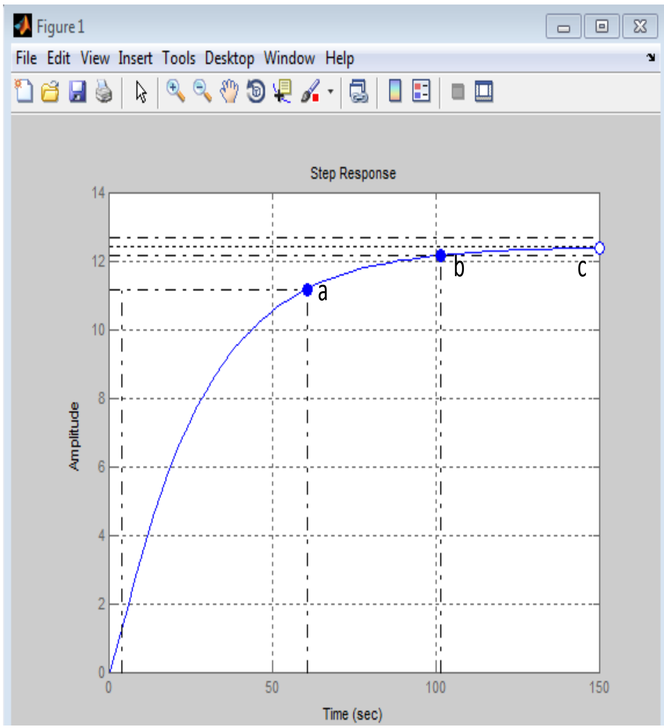



Figure 4:Smith Predictor Step Response of System.

ITEE Journal
Information Technology & Electrical Engineering

ISSN: - 2306-708X



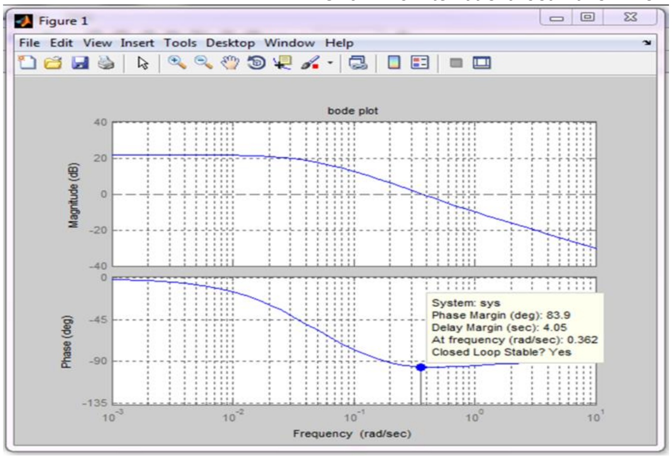



**Figure 5:**Phase and Magnitude Response of the Boiler system

## 3.1. SIMULATION RESULTS FOR PAD'E APPROXIMATION

### (i) PID METHOD

Fig. 6 shows the Smith Predictor Simulation diagram For PID Controller uses for Pad'e Approximation. **Table 1** lists the Transient responses and corresponding gains. System responses for different delays for Pad'emodelling using PID controller is shown in Figure6.

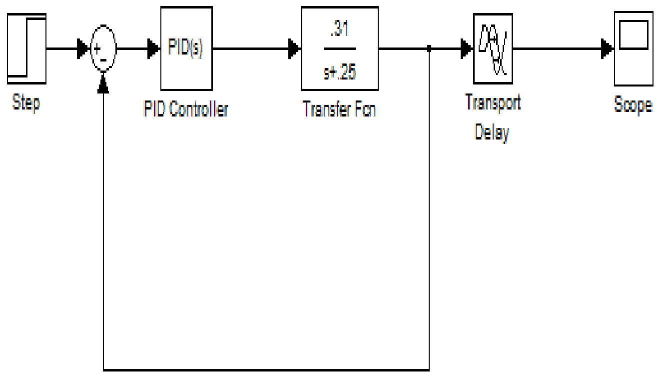



Figure 6:Smith Predictor Simulation diagram For PID Controller (Pad'e approximation).

**Table 1**

*Transient responses and corresponding gains for Pad'emodelling using PID controller*

| Delay (h) | Rise time | Settling time | Over shoot | Peak | Gain | | |
|---|---|---|---|---|---|---|---|
| | | | | | $K_p$ | $K_i$ | $K_d$ |
| 0.0 | 19.1 | 60.4 | 7.45 | 1.07 | 0.1318 | 0.0066 | 0.4964 |
| 1.0 | 20.4 | 62.7 | 9.17 | 1.09 | 0.1430 | 0.0066 | 0.9380 |
| 2.0 | 19.6 | 61.5 | 8.51 | 1.08 | 0.1463 | 0.0667 | 0.2730 |
| 3.5 | 9.39 | 66 | 4.47 | 1.04 | 0.0253 | 0.0919 | 0.0000 |
| 5.0 | 4.12 | 67.8 | 10.7 | 1.11 | 0.7143 | 0.1055 | 0.5563 |

### (ii) POLE-PLACEMENT METHOD BY ARE

Algebraic Riccati equation (ARE) approach is used to achieve optimal state feedback gain. For the given system specifications, the poles should be within a specified region. The system matrix A is modified to achieve this constraint before it is passed to ARE [7].

$$\frac{(A-\alpha I)}{r}P\frac{(A-\alpha I)'}{r}-P-\frac{(A-\alpha I)'}{r}PB\left(r^2R+B'PB\right)^{-1}B'P\frac{(A-\alpha I)}{r}=Q$$

(23)

Where, P is a positive definite symmetric solution of the Riccati equation, and the state feedback law to assign all the closed-loop poles of system in the left half plane. **Table 2** Transient responses and corresponding gains using Pole-placement method. System responses for different delays for Pad'emodelling using Pole placement method is shown in Figure7.

Table 2

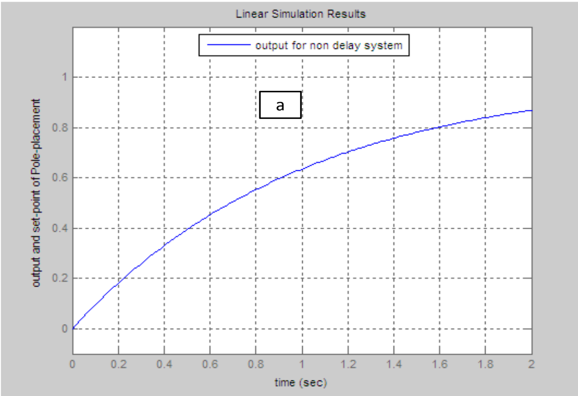
Transient responses and corresponding gains using Pole-placement method (Pad'emodelling)

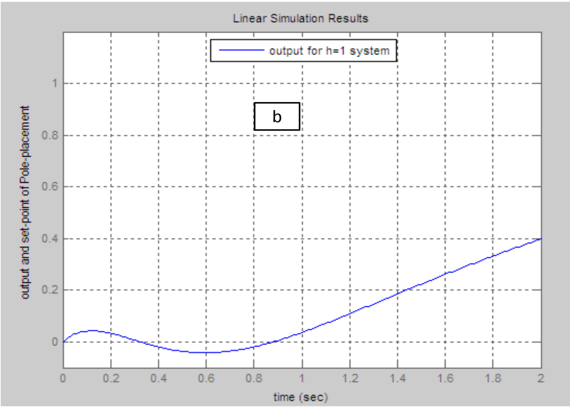
| Delay (h) | α | Closed Loop Poles | Gains |
|---|---|---|---|
| 0.0 | 1.0 | -1.1501+0.0409i | K=0.3405 |
| | | -1.1501-0.0409i | =0.0950 |
| 1.0 | 0.5 | -3.0000+1.7321i | K=0.3405 |
| | | -3.0000-1.7321i | =2.1379 |
| | | -0.2952+0.1271i | =4.6560 |
| | | -0.2952-0.1271i | =1.1404 |
| 2.0 | 0.5 | 0.6997 | K=0.0239 |
| | | -0.5649 | =0.0417 |
| | | -2.0046+0.8525i | =0.0557 |
| | | -2.0046-0.8525i | =0.0139 |
| 3.5 | 0.5 | -0.6992 | K=0.0180 |
| | | -0.5545 | =0.0035 |
| | | -1.3643+0.4653i | =0.0041 |
| | | -1.3643-0.4653i | =0.0023 |
| 5.0 | 0.5 | -0.6786 | K=0.0190 |
| | | -0.5684 | =0.0107 |
| | | -1.1110+0.3003i | =0.0005 |
| | | -1.1110-0.3003i | =0.0017 |

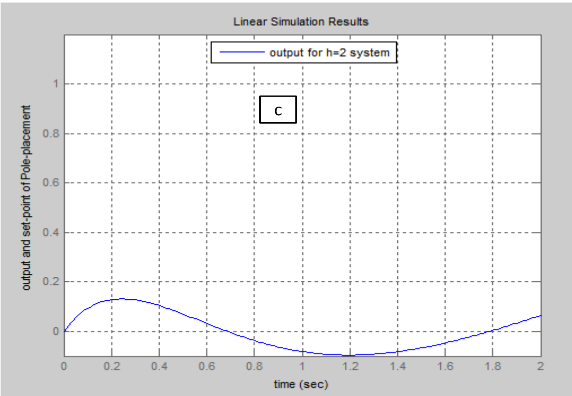
ITEE Journal
Information Technology & Electrical Engineering

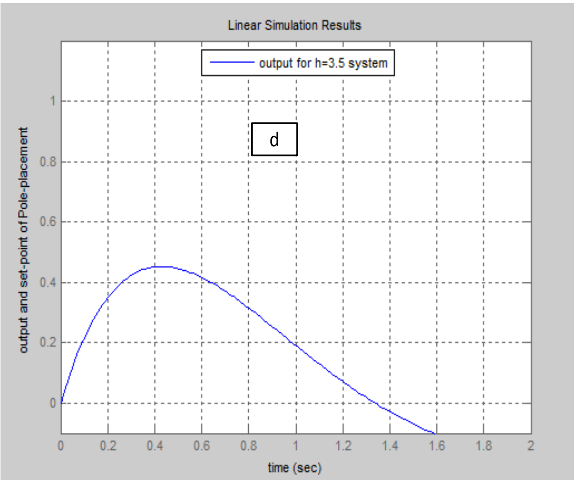
ISSN: - 2306-708X





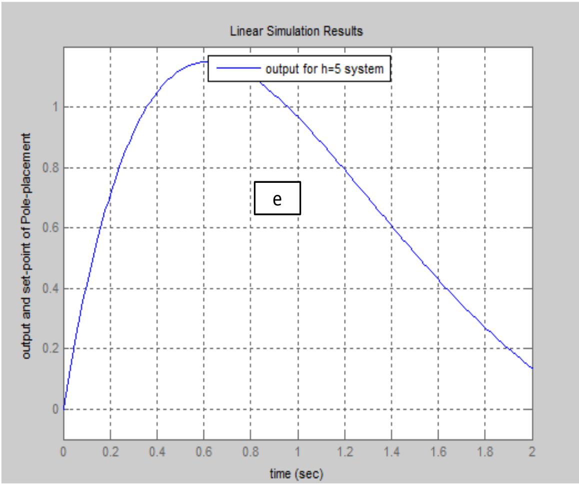




Figure7: System responses for different delays for Pad'emodelling using pole placement method (a) Non delayed system, (b) Delay h = 1s, (c) Delay h = 2s, (d) Delay h = 3.5s, (e) Delay h =5s.

### 3.2. SIMULATION RESULTS FOR SMITH PREDICTOR

### (I) PID METHOD

Figure8 shows the Smith Predictor Simulation diagram. **Table 3** lists the Transient responses and corresponding gains for different delays. System responses for different delays for Smith Predictor modelling using PID method for different delay is shown in Figure 9.

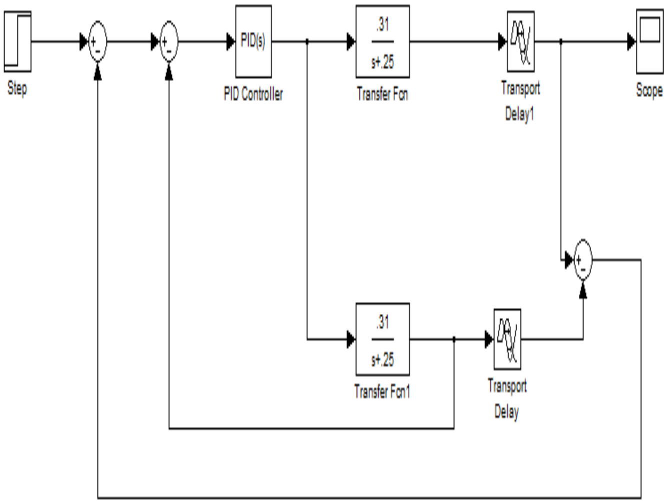



Figure8: Simulation Diagram of Boiler System (Smith Predictor)

Volume 5, Issue 5
October 2016

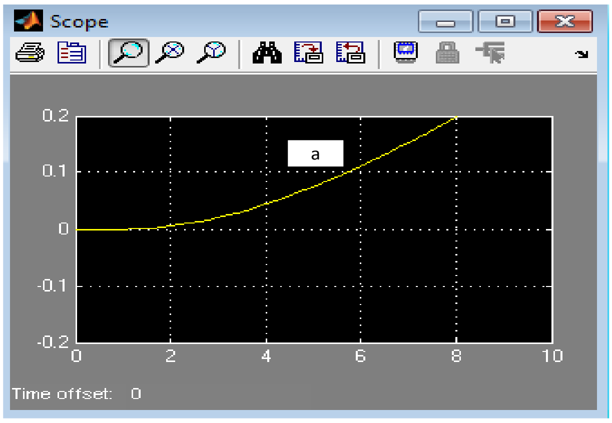
ITEE Journal
Information Technology & Electrical Engineering

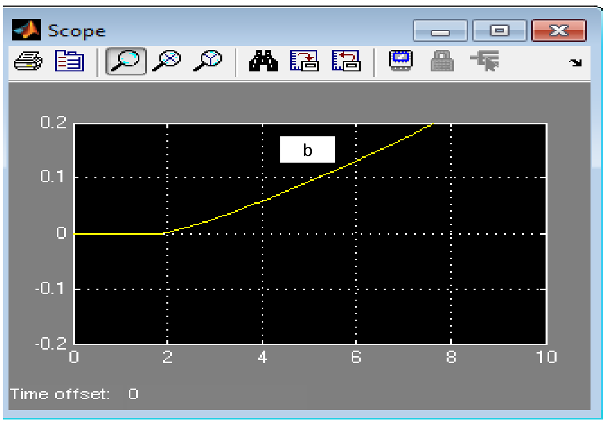
ISSN: - 2306-708X

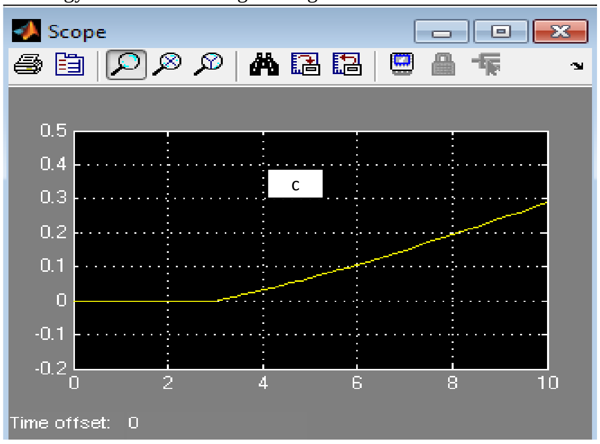


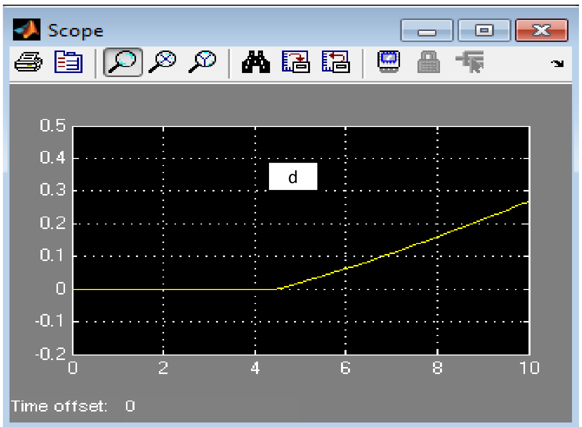
**Table 3**
Transient responses and corresponding gains for different Delays

| Delay (h) | Rise time | Settling time | Overshoot | Peak | Gain | | |
|---|---|---|---|---|---|---|---|
| | | | | | $K_p$ | $K_i$ | $K_d$ |
| 0.0 | 19.1 | 60.4 | 7.45 | 1.07 | 0.115 | 0.007 | 0.458 |
| 1.0 | 20.7 | 63.0 | 8.73 | 1.09 | 0.143 | 0.006 | 0.847 |
| 2.0 | 18.2 | 62.7 | 8.99 | 1.09 | 0.149 | 0.007 | 0.189 |
| 3.5 | 12.7 | 48.5 | 5.23 | 1.05 | 0.162 | 0.006 | 0.295 |
| 5.0 | 21.1 | 67.9 | 6.05 | 1.06 | 0.099 | 0.003 | 0.000 |



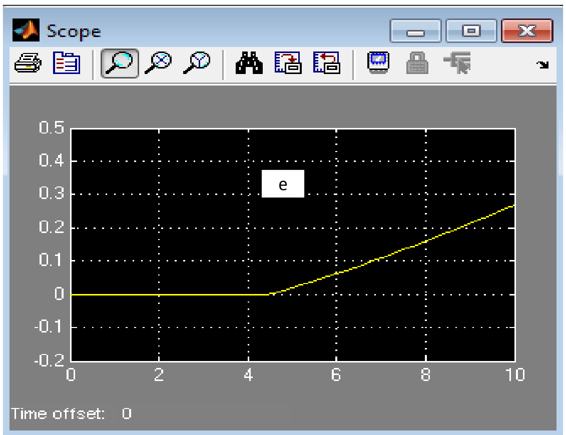




Figure9: System responses for different delays for Smith Predictor modelling using PID method (a) Delay h=0s, (b) Delay h = 1s, (c) Delay h = 2s, (d) Delay h = 3.5s, (e) Delay h =5

### (ii) POLE PLACEMENT METHOD BY ARE

**Table 4** lists the Transient responses and corresponding gains for different delays using Pole placement method by ARE. System responses for different delay using Pole placement method are shown in Figure 10.

Volume 5, Issue 5
October 2016

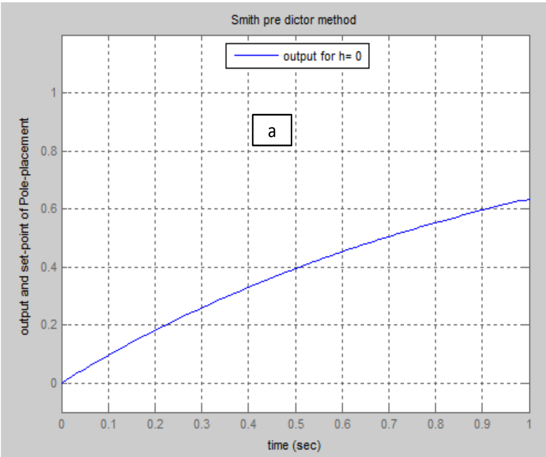
ITEE Journal
Information Technology & Electrical Engineering

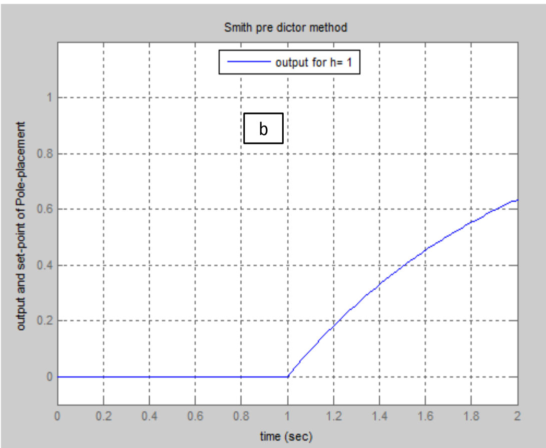
ISSN: - 2306-708X

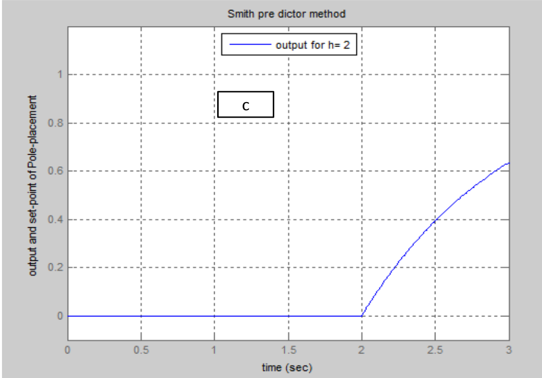


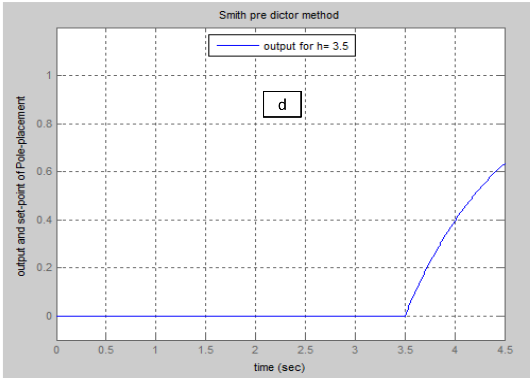
Table 4
Transient responses and corresponding gains for different delays

| Delay (h) | α | Closed Loop Poles | Gains |
|---|---|---|---|
| 0.0 | 1.0 | -1.1501+0.0409i | K=0.3405 |
| | | -1.1501-0.0409i | =0.7603 |
| 1.0 | 0.5 | -3.0000+1.7321i | K=0.3405 |
| | | -3.0000-1.7321i | =0.7603 |
| 2.0 | 0.5 | 0.6997 | K=0.3405 |
| | | -0.5649 | =0.7603 |
| 3.5 | 0.5 | -0.6992 | K=0.3405 |
| | | -0.5545 | =0.7603 |
| 5.0 | 0.5 | -1.1110+0.3003i | K=0.3405 |
| | | -1.1110-0.3003i | =0.7603 |



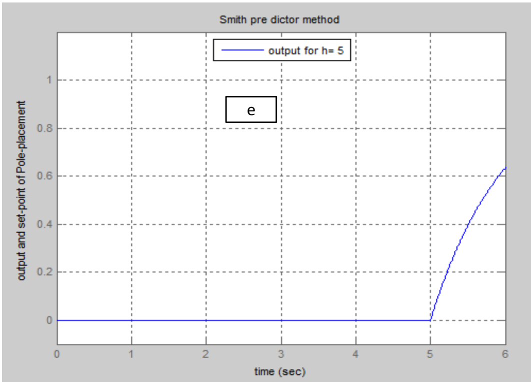




Figure 10: System responses for different delays for Smith Predictor modelling using pole-placement method (a) Delay h = 0s, (b) Delay h = 1s, (c) Delay h = 2s, (d) Delay h =3.5s, (e) Delay h =5.s

## 4. CONCLUSION

A mathematical model of Boiler system was developed by using block reduction method in this work. The level control in boiler system is done by controlling the delay using Pad'e and Smith predictor method. The analysis is carried out by using Transient Response and Frequency response Methods. A comparison of different methods for compensation of delayed systems has also discussed. The delay has been modelled using Smith predictor and Pad'e approximation. In each case, the controller was designed using both ARE and PID approaches. The simulation results show the smith predictor method gave better results in comparison to the Pad'e approximation method. Also ARE approach gives better performance rather than PID methods. Hence, it can be concluded that Smith predictor

ITEE Journal
Information Technology & Electrical Engineering

ISSN: - 2306-708X



combined with ARE for modeling, the delay and designing the controller gives the optimal performance.

## REFERENCES

[1] A. B. Ali and T. Malayan, Digital Control Design for the Boiler Drum (QAD Model BDT921), Final Project, KUiTHO, 2005.

[2] D. S. Rana, and Sudha Rani, Fuzzy Logic Based Control System for Fresh WaterAquaculture: A MATLABbased Simulation Approach, SERBIAN JOURNAL OF ELECTRICAL ENGINEERING, 12, pp.171-182, 2015.

[3] J.P. Richard, Time-delay Systems: An Overview of Some Recent Advances and Open Problems, Automatica, 39:pp.1667–1694, 2003.

[4] F.G. Shinskey, Process Control Systems, McGraw-Hill Book Company, 1967.

[5] T.Furukawa and E. Shimemura, Predictive Control for Systems with Time Delay," International Journal of Control, 37: 399–412, 1983.

[6] J. Hench, C. He, V. Kuˇcera, and V. Mehrmann, dampening Controllers via a Riccati EquationApproach, IEEE TRANSACTIONS ON AUTOMATIC CONTROL, 43, 1998.

[7] K. Ogata; Modern Control Engineering, 3rd Edition, PHI.

[8] Bittanti, Laub, and Willems, TheRiccati Equation, Springer Verlag, 1991.

[9] R. Sharma, D.S. Rana, A.M Gaur: A Fuzzy Logic based Automatic Control of Rotary Crane (A Simulation Approach), Advanced Materials Research, 403-408, pp. 4659 – 4666, 2011.

## AUTHOR PROFILES

**Dinesh Singh Rana** is currently working as Professor in department of Instrumentation (Formerly Institute of Instrumentation Engineering). He received his M.Tech. degree in Instrumentation from National Institute of Technology (Formerly R.E.C) Kurukshetra and Ph.D. in Instrumentation Engineering from Kurukshetra University. He has 2 years industrial experience and 20 years teaching and research experience. He joins Instrumentation Department of Kurukshetra University in 1995 and worked as Coordinator of M.Tech. in Instrumentation Engineering to promote TEQIP programme in the Kurukshetra University. He has supervised M.Tech. and Ph.D. theses in large number of students and published more than 70 research papers in journals of international repute. He has attended and presented papers several national & international conferences of repute. He is the member of various professional bodies/Societies of repute. His area of research and interest includes Process Measurement & control, Material development for Smart sensors/MEMS design, Modeling & Automation (PLC, DCS, Embedded &Neuro-Fuzzy controllers design & SCADA), and Digital Signal processing

**DikshaSoni** received his M.Tech from Department of Instrumentation, Kurukshetra University, Kurukshetra in 2014. Earlier she has attained B.Techin Instrumentation Engineering from JMIT, Radaur, Yamunanagar. Now she is working in Expedia as an Incident Response Analyst. Her areas of interest includes Process control, modeling and controller design.