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Performance Comparison of Cross and Rectangular Quadrature Amplitude Modulation over Nakagami Fading Channels for Equal Energy Case

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ABSTRACT

In this paper SEP (symbol error probability) performance of Cross and rectangular QAM is compared over Nakagami fading channels for EEC (Equal Energy Case). The SEP expressions are derived using MGF (moment generating function) approach and are composed of linear sum of integrals that have integrands that are exponential and trigonometric functions. SEP expressions of cross and rectangular QAM are considered for AWGN channel that only contains Q-functions. Equal Energy Case states that Euclidean distance ‘d’ between symbols points of rectangular QAM, when reduced provides comparable average energy with that of cross QAM but still it would not affect the SEP performance of rectangular QAM. Cross QAM provides considerably better performance and can obtain gain of at least 1.1 dB over rectangular QAM, when SEP is less than 0.3 in Nakagami fading channels. The better performance is due to the cross shaped constellation structure of cross QAM.

Keywords:*MGF (moment generating function), SEP (symbol error probability), QAM (quadrature amplitudemodulation), EEC (Equal Energy Case).*

1. INTRODUCTION

QAM is a preferred modulation scheme for digital communication over fading channels due to its high bandwidth efficiency. In ASK (amplitude shift keying) the transmitted signal an experience deep fades due to channel’s fading amplitude and white noise. Similarly in PSK (phase shift keying) for larger constellation size ‘M’ the phase changes in carrier signal are very complex and a very expensive receiver is required. To provide tradeoff between performance and complexity a Hybrid scheme is commonly used that is known as QAM (quadrature amplitude modulation). When transmission of even bits per symbol is the requirement then square QAM is preferred constellation but when odd bits per symbol is required then both rectangular and cross QAM (XQAM) can be considered. Cross QAM is usually the better choice as it is an energy efficient scheme. Cross QAM has lesser average and peak energy as compared to Rectangular QAM (RQAM). In other words we can say that to achieve a particular SEP cross QAM require lesser SNR in dBs then that required by the Rectangular QAM.

Despite of the higher efficiency of XQAM, the derivation of average SEP expression has many complications as compared to that of the rectangular QAM. One of the important reasons for complex calculations is that the cross QAM has inter dependent inphase and quadrature components unlike rectangular QAM. Rectangular QAM on the other hand can be considered as the composition of two PAM (pulse amplitude modulation) signals and can be easily demodulated.

Cross QAM have many applications such as adaptive modulation scheme, where the constellation size ‘M’ is improved according to the channel behavior. When channel quality is good, the constellation size is increased by ‘k+1’ bits per symbol. If we are to consider only even bits per symbol (square QAM) then the increment size would be ‘k+2’ bits per symbol (we have to go from 16 to 64 to 256 QAM...). XQAM however provides the opportunity to reduce the intermediate step size from ‘k+2’ to ‘k+1’ bits per symbol (we need to go from 16 to 32 to 64 QAM…). Use of XQAM for single bit increase makes the change relatively smoother and enables the system to perform better over a required data rate. Cross QAM with symbol length of 5 to 15 bits are commonly used in ADSL and VDSL applications[1-2], also 32 and 128 XQAM had been applied in digital video broadcasting [3], XQAM also has many application regarding blind equalization, where the channel response is estimated by the equalizer without the training sequence [4].

[5] derived average SEP expressions for XQAM in AWGN and fading channels including Rayleigh and Nakagamichannels. The expressions derived contain finite integrals with integrands that are exponential and trigonometric functions.In [6] we have mathematical models of various multipath radio channels that are useful for performance analysis of different digital modulation schemes.

In [7-9] exact average SEP expressions for 32,128 and 512 cross QAM had been derived for AWGN and Rayleigh fading channel. In [10] closed form expression for rectangular QAM had been presented for AWGN and Rayleigh channels. In [11] exact expression for BER (bit error rate) of cross QAM had been derived in AWGN and Rayleigh channel by considering the contribution of each bit individually and along with that Smith’s approximation for BER of cross QAM is also discussed.

In this paper, we compare SEP performance of cross and rectangular QAM in Nakagami fading channels for EEC. The remaining sectionsare arranged as follows; section (II) describes constellation structures of both modulation schemes and also compares them by considering multiple parameters. In section (III) we describe channel models by considering the probability density functions (PDFs) and moment generating

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functions (MGFs). In section (IV) we consider SEP expressions for cross and rectangular QAM in AWGN channel. In section (V) SEP expressions for Nakagami fading channels are derived using MGF approach. Section (VI) present numerical results and Section (VII) provides conclusion.

2. CONSTELLATION STRUCTURES AND PARAMETERIC COMPARISON

Constellation structure of cross and rectangular QAM are slightly different and according to [7] we can construct constellation shape of cross QAM from rectangular QAM by shifting $\frac{\sqrt{2M}}{8}$ columns on either side to top and bottom positions. Fig. 1 (a) and (b) shows constellation structures for 32-XQAM and 32-RQAM respectively.

In both cross and rectangular QAM the constellation structure includes three types of symbol points: interior, edge and corner points. The decision regions for interior and edge points are square and semi-infinite rectangular in shape, while for corner points decision regions are comparatively different as they are made up of horizontal, vertical and 45 degree lines. In rectangular QAM number of corner points remains fixed while in case of cross QAM their number vary with increase in constellation size 'M'.

For comparison of both modulation schemes we consider three parameters: $\bar{b}$ (bits per dimension), $\overline{Ex}$ (average energy per dimension) and CFM (constellation figure of merit).

We define,

$$M = 2^k, \quad \text{where k = 5, 7, 9, 11, ….}$$

Three parameters considered for comparison are defined as,

$$\bar{b} = k/N \quad (1)$$

$$\overline{Ex} = Eav/N \quad (2)$$

$$CFM = \frac{\left(\frac{d}{2}\right)^2}{\overline{Ex}} \quad (3)$$

Where 'N' are the number of dimensions, in case of QAM N=2, while 'Eav' is the average energy that depends on the symbol points spacing 'd' and constellation size 'M'. Average energy for both cross and rectangular QAM is given by,

<u>Cross QAM :</u> $E\text{av} = \frac{d^2}{6}\left(\frac{31M}{32} - 1\right) \quad (4)$

<u>Rectangular QAM :</u> $E\text{av} = \frac{d^2}{6}\left(\frac{10M}{8} - 1\right) \quad (5)$

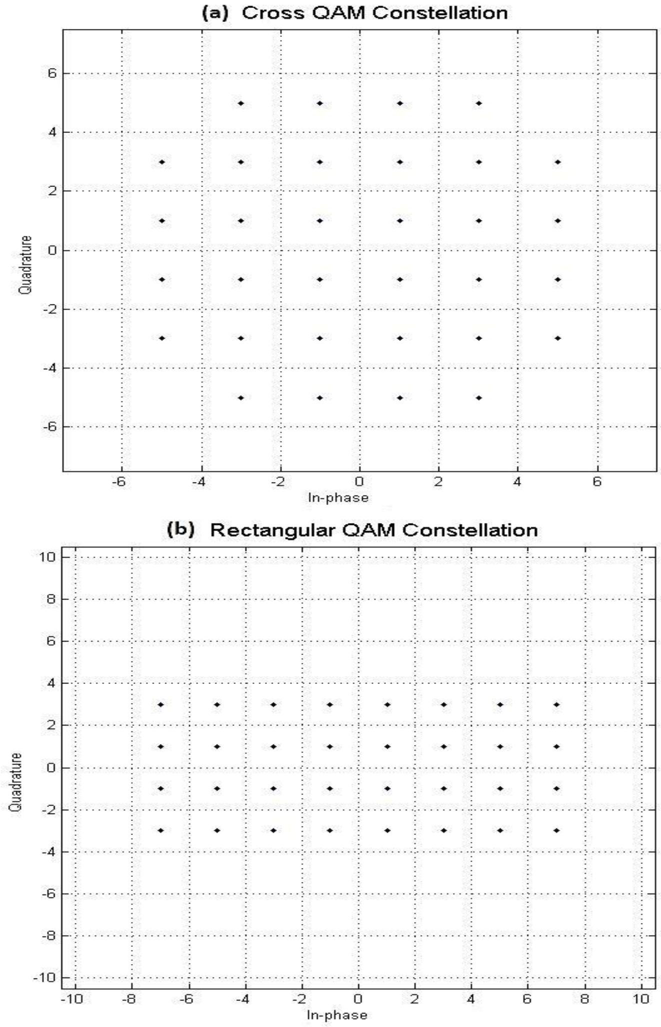

Fig.1 (a) Cross 32-QAM (b) Rectangular 32-QAM.

Table.1 compares cross and rectangular QAM modulation schemes with the help of three parameters that are defined previously for various constellation sizes (taking d=2).

Constellation Size (M)	Cross QAM			Rectangular QAM		
	$\bar{b}$	$\overline{Ex}$	CFM	$\bar{b}$	$\overline{Ex}$	CFM
32	2.5	10.33	0.0968	2.5	13	0.0769
128	3.5	42.33	0.0236	3.5	53	0.0189
512	4.5	170.33	0.0059	4.5	213	0.0047

Table.1

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3. SYSTEM MODEL

A. Probability Density Functions (PDFs)

The Nakagami-m and Nakagami-q channels are described with the help of their fading parameters 'm' and 'q' respectively. The probability density functions (PDFs) of the Nakagami-m and Nakagami-q channels as a function of channel fading amplitude 'ß' as described in [6] are given as,

$$P_m(ß) = \frac{2m^m ß^{2m-1}}{\Omega^m\, \Gamma(m)} exp\left(-\frac{mß^2}{\Omega}\right),\ ß \geq 0 \quad (6)$$

$$P_q(ß) = \frac{(1+q^2)ß}{q\,\Omega} exp\left(-\frac{(1+q^2)^2 ß^2}{4q^2\,\Omega}\right) I_0\left(\frac{(1-q^4)ß^2}{4q^2\,\Omega}\right),\ ß \geq 0 \quad (7)$$

Where 'Ω' is the average fading power and $I_0(.)$ is the modified Bessel function of zero order and first kind. Fading parameters 'm' and 'q' ranges between '$\frac{1}{2}$ to ∞' and '0 to 1' respectively. Fig. 2 shows the PDFs of both fading channels as a function of fading amplitude 'ß' for various fading parameters.

Using change of variables, we can derive PDF expressions for both Nakagami-m and Nakagami-q channels as a function of instantaneous SNR 'γ' that are given as,

$$P_m(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m\, \Gamma(m)}\ exp\left(-\frac{m\gamma}{\bar{\gamma}}\right),\quad \gamma \geq 0 \quad (8)$$

$$P_q(\gamma) = \frac{(1+q^2)}{2q\,\bar{\gamma}} exp\left(-\frac{(1+q^2)^2 \gamma}{4q^2\bar{\gamma}}\right) I_0\left(\frac{(1-q^4)\gamma}{4q^2\bar{\gamma}}\right),\ \gamma \geq 0 \quad (9)$$

B. Moment Generating Functions (MGFs)

Relating to the Probability density functions (PDFs) in (8) and (9), we can determine the Moment Generating Functions (MGFs) in terms of average fading SNR '$\bar{\gamma}$' by taking Laplace transform with reversed sign as in [6],

$$M_{m\text{-}\gamma}(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m},\ m \geq 1/2 \quad (10)$$

$$M_{q\text{-}\gamma}(s) = \left(1 - 2s\bar{\gamma} + \frac{(2s\bar{\gamma})^2 q^2}{(1+q^2)^2}\right)^{-1/2},\ 0 \leq q \leq 1 \quad (11)$$

4. SEP OF CROSS AND RECTANGULAR QAM IN AWGN CHANNEL

Cross QAM as described previously is an energy efficient version of QAM that is preferred when we areconsidering odd bits per symbol. SEP expressions of cross and rectangular QAM in AWGN channel in terms ofEuclidean distance 'd' betweensymbol points and average energy 'Eav' are given by,

$$P_{XQAM}(\gamma) = \frac{1}{M}\begin{bmatrix} M\left(4M - 6\sqrt{\frac{M}{2}}\right)Q\left(\sqrt{2\alpha\gamma}\right) + 4Q\left(2\sqrt{\alpha\gamma}\right) \\ -\left(4M - 12\sqrt{\frac{M}{2}} + 12\right)Q\left(\sqrt{2\alpha\gamma}\right)^2 \end{bmatrix} \quad (12)$$

Where,

$$\alpha = \frac{d^2}{4Eav}$$

$$P_{RECTQAM}(\gamma) = \frac{15}{4}\left[1 - \frac{64}{15M}\right]Q\left(\beta\sqrt{\gamma}\right) - \frac{7}{4}\left[1 - 8MQ\beta\gamma 2\right. \quad (13)$$

Where,

$$\beta = \frac{d}{\sqrt{2Eav}}$$

In (12) and (13) 'M' is constellation size while 'γ' is the instantaneous SNR.

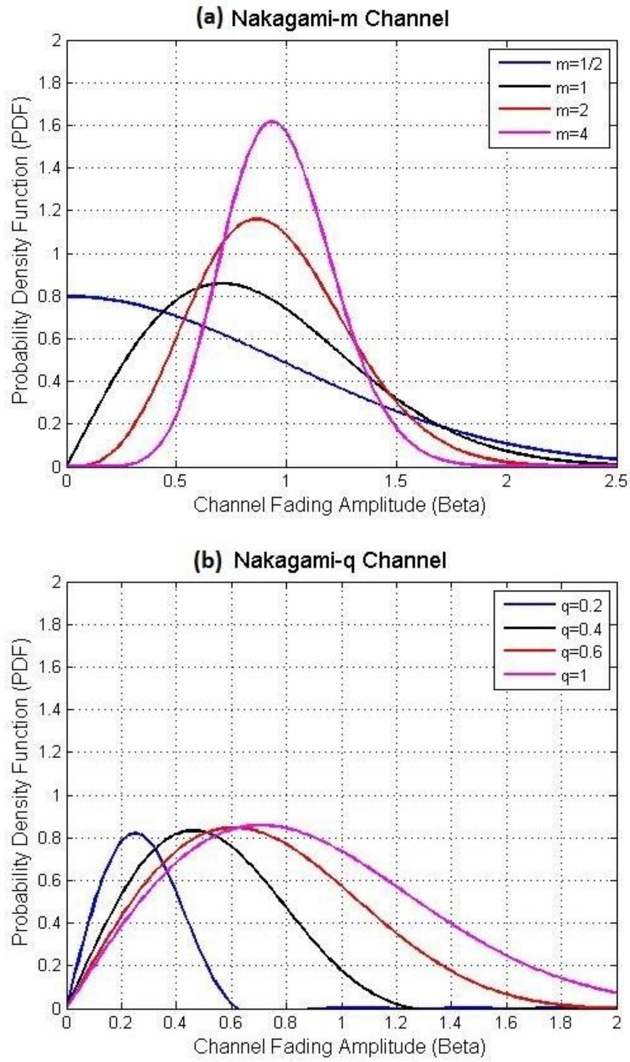

Fig. 2 (a) PDF of Nakagami-m channel, m = [1/2 1 2 4]
(b)PDF of Nakagami-q channel, q = [0.2 0.4 0.6 1].

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5. SEP OF CROSS AND RECTANGULAR QAM INNAKAGAMI FADING CHANNELS

SEP expressions for cross and rectangular QAM in Nakagami-m and Nakagami-q fading channels can be derived by averaging the SEP performance of both modulation schemes over the PDF of Nakagami-m and Nakagami-q fading distributions.

$$P_e = \int_0^\infty P_{QAM}(\gamma) P(\gamma)\, d\gamma \quad (14)$$

$P_{QAM}(\gamma)$ is the SEP expression of cross or rectangular QAM in AWGN channel given by (12) and (13), also$P(\gamma)$ is PDF of Nakagamifading channel in terms of instantaneous SNR 'γ'. Evaluating (14) we have linear sum of one dimensional integrals that has infinite limits. We need to reduce these integrals to an appropriate form using specific form of Q-function that has finite limits, so that the SEP expression in (14)has higher numerical computational accuracy.

We define,

$$S_1(a) = \int_0^\infty Q(a\sqrt{\gamma})\, P(\gamma)\, d\gamma \quad (15)$$

$$S_2(a) = \int_0^\infty Q(a\sqrt{\gamma})^2\, P(\gamma)\, d\gamma \quad (16)$$

Where 'a' is a constant that depends on the specific type modulation/detection process used. we consider modified form of Q-function that is defined by,

$$Q(v) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} exp\left(-\frac{v^2}{2\sin\theta^2}\right)\, d\theta \quad (17)$$

MGF is defined as,

$$M_\gamma(S) = \int_0^\infty e^{s\gamma} P(\gamma)\, d\gamma \quad (18)$$

Therefore using (17) and (18), we can transform (15) and (16) in terms of MGF as,

$$S_1(a) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma\left(-\frac{a^2}{2\sin\theta^2}\right)\, d\theta (19)$$

$$S_2(a) = \frac{1}{\pi}\int_0^{\frac{\pi}{4}} M_\gamma\left(-\frac{a^2}{2\sin\theta^2}\right)\, d\theta (20)$$

Using (14), (19) and (20), we have average SEP expressions for both M-ary XQAM and RQAM in Nakagami fading channels that are given by,

Cross QAM:

$$P_e = \frac{1}{M}\left[M\left(4M - 6\sqrt{\frac{M}{2}}\right)S_1(\sqrt{2\alpha}) + 4S_1(2\sqrt{\alpha}) - \right.$$

$$4M - 12M2 + 12S22\alpha (21)$$

Rectangular QAM:

$$P_e = \frac{15}{4}\left[1 - \frac{64}{15M}\right]S_1(\beta) - \frac{7}{4}\left[1 - \frac{8}{M}\right]S_2(\beta) (22)$$

Where 'M' is the constellation size and 'S_1'and'S_2' are defined in (19) and (20).

According to Equal Energy Case (EEC) the average energy of rectangular QAM can be made comparable with that of cross QAM by reducing the symbol point spacing 'd', but bringing the symbol points closer doesn't affect the SEP performance of rectangular QAM as decrease in 'd' also causes decrease in average energy keeping the ratio 'β' approximately constant.

6. NUMERICAL RESULTS

Fig. 3 shows SEP performance comparison of 32-XQAM and 32-RQAM in Nakagami-m fading channel. We can observe that cross QAM has considerably better performance as compared to rectangular QAM, also increase in fading parameter 'm' causes improvement in system performance due to lesser fades. Reducing 'd' doesn't affect the SEP performance of rectangular QAM as discussed previously. Fig. 4 shows SEP performance comparison of both modulation schemes in Nakagami-q fading channel for various values of fading parameter 'q'. Also it can be seen that SNR required to achieve particular SEP in Cross 32-QAM is comparatively lesser than that of rectangular 32-QAM.

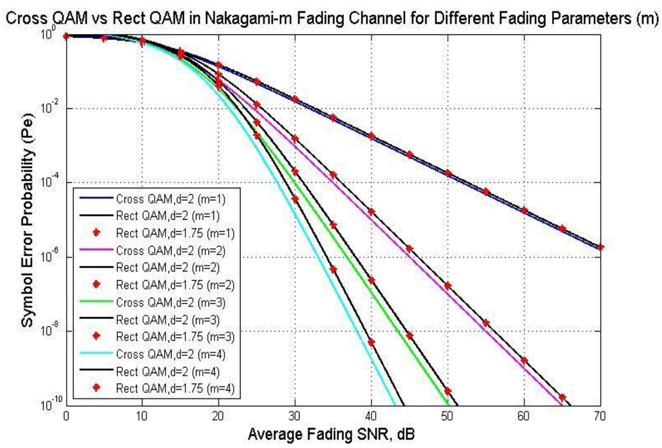

Fig.3 Average SEP comparison of 32-XQAM and 32-RQAM in Nakagami-m fading channel for EEC.

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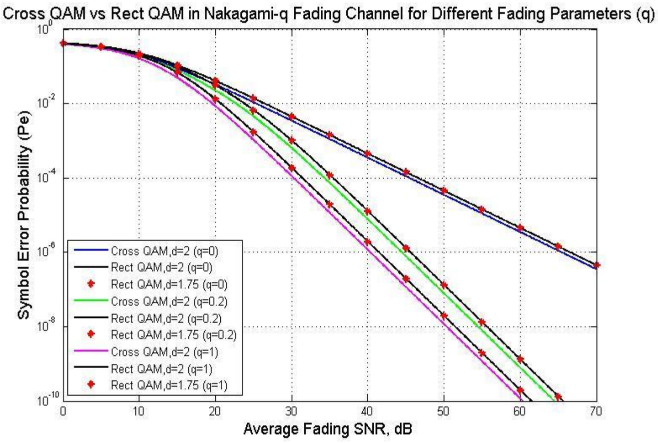

Fig.4 Average SEP comparison of 32-XQAM and 32-RQAM in Nakagami-q fading channel for EEC.

7. CONCLUSION

Previously average SEP expressions for M-ary cross and rectangular QAM in AWGN and multiple fading channels have been reported. In this paper, average SEP performance of M-ary cross QAM is compared with rectangular QAM for EEC in Nakagami fading channels for various values of fading parameters. It can be noted that cross QAM has better performance in both channels due to its unique cross shaped constellation structure.

REFERENCES

[1] Asymmetric Digital Subscriber Line (ADSL) Transceivers, ITU-T Std. G.992.1, Jun. 1999.

[2] Very High Speed Digital Subscriber Line Transceivers, ITU-T Std. G.993.1, Jun. 2004.

[3] Digital Video Broadcasting (DVB); Framing Structure, Channel Coding and Modulation for Cable Systems, ETSI Std. EN 300 429, Apr. 1998.

[4] S. Abrar and I. M. Qureshi, "Blind equalization of cross-QAM signals," *IEEE Signal Processing Letters*, vol. 13, no. 12, pp. 745-748, 2006.

[5] Xi-chun Zhang, Hua Yu and Gang Wei, "Exact Symbol Error Probability of Cross-QAM in AWGN and Fading Channels," vol. 2010, Article ID 917954. [doi:10.1155/2010/917954]

[6] M.K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels, A Unified Approach to Performance Analysis*, John Wiley & Sons: New York, 2000.

[7] N. C. Beaulieu and Y. Chen, "Closed-form expressions for the exact symbol error probability of 32-cross-QAM in AWGN and in slow Nakagami fading," *IEEE Communications Letters*, vol. 11, no. 4, pp. 310-312, 2007.

[8] J. Li, X. D. Zhang, and N. C. Beaulieu, "Precise calculation of the SEP of 128- and 512-Cross-QAM in AWGN," *IEEE Communications Letters*, vol. 12, no. 1, pp. 1-3, 2008.

[9] H. Yu and G. Wei, "Symbol error probability of cross QAM in Rayleigh fading channels," *IEEE Communications Letters*, vol. 14, no. 5, Article ID 5456037, pp. 375-377, 2010.

[10] N. C. Beaulieu, "A useful integral for wireless communication theory and its application to rectangular signaling constellation error rates," *IEEE Transactions on communications*, vol. 54, no. 5, pp. 802-805, 2006.

[11] P. K. Vitthaladevuni, M. –S. Alouini, and J. C. Kieffer, "Exact BER computation for cross QAM constellations," *IEEE Transactions on Wireless Communications*, vol. 4, no. 6, pp. 3039-3050, 2005.

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