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Vehicle Tracking Using Kalman Filter Variants

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ABSTRACT

Extended Kalman and Unscented Kalman filter are used to track the position and velocity of Vehicle moving in a nominal given direction at nominal Speed. The measurements are noisy version of noise and Bearing. The filters are used to nullify the effect of noise and track the vehicle with true position and velocity. The results of both the filters are compared and analyzed and conclusion are made that which filter works best in the given noise scenario.

Keywords: Vehicle tracking, ExtendedKalman, Unscented Kalman

1. INTRODUCTION

In arriving at a model for the dynamics of the vehicle we assume a constant velocity model, perturbed only by wind gusts, slight speed correction, etc might occur in an aircraft. We model these perturbations as noise inputs, so that the velocity components in the x and y directions at time n are:

$$v_x[n] = v_x[n-1] + u_x[n] \qquad (1)$$
$$v_y[n] = v_y[n-1] + u_y[n] \qquad (2)$$

Without the noise perturbations $u_x[n]u_y[n]$ the velocities would be constant, and hence the vehicle would be modeled as traveling in a straight line. From the equation of motion the position at time n is:

$$r_x[n] = r_x[n-1] + v_x[n]\Delta \qquad (3)$$
$$r_y[n] = r_y[n-1] + v_y[n]\Delta \qquad (4)$$

Where Δ is the time interval between samples, in the discretized model for the equations of motion the vehicle is modeled as moving at the velocity of the previous time instant and then changing abruptly at the next time instant, an approximation to the true continuous behavior. We considered thesignal vector as having the attribute of the position and velocity components.

$$s[n] = \begin{bmatrix} r_x[n] \\ r_y[n] \\ v_x[n] \\ v_y[n] \end{bmatrix} \qquad (5)$$

$$\begin{bmatrix} r_x[n] \\ r_y[n] \\ v_x[n] \\ v_y[n] \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x[n-1] \\ r_y[n-1] \\ v_x[n-1] \\ v_y[n-1] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_x[n] \\ u_y[n] \end{bmatrix} \qquad (6)$$

$$A = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (7)$$

The measurements are noise observations of range and bearing

$$R[n] = \sqrt{r_x^2[n] + r_y^2[n]} \qquad (8)$$
$$\beta[n] = \arctan\frac{r_y[n]}{r_x[n]} \qquad (9)$$

Or

$$\hat{R}[n] = R[n] + W_R[n] \qquad (10)$$
$$\hat{\beta}[n] = \beta[n] + W_\beta[n] \qquad (11)$$

Generally the observation equation is

$$x[n] = h(s[n]) + w[n] \qquad (12)$$

Where h is the function

$$h(s[n]) = \begin{bmatrix} \sqrt{r_x^2[n] + r_y^2[n]} \\ \arctan\frac{r_y[n]}{r_x[n]} \end{bmatrix} \qquad (13)$$

Unfortunately, the measurement vector is nonlinear in the signal parameters. To estimate the signal vector we will need to apply extended and unscented Kalman filter.

2. TRAJECTORY OF VECHILE

As described in [2] Vehicle trajectory is modeled as:

$$r_x[n] = 10 - 0.2n \qquad (14)$$
$$r_y[n] = -5 + 0.2n \qquad (15)$$

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Ideal Trajectory and true trajectories are shown in figures true trajectories are obtained by adding plant and measurement noises. And this model is then initialized with filters to track ideal trajectories in presence of noise.

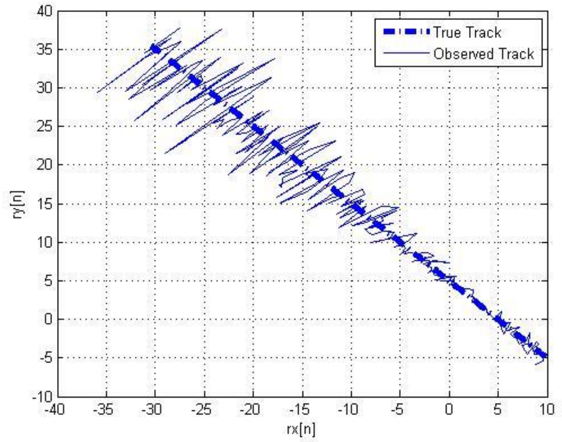

Fig. 1: True and Observed track of vehicle moving in a given direction at constant speed

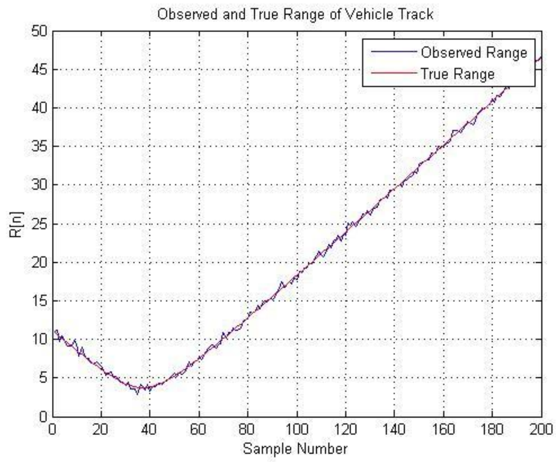

Fig. 2: Ideal and Observed Range

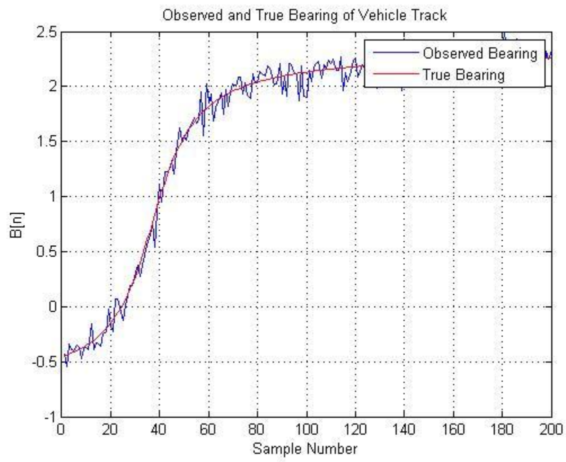

Fig. 3: Ideal and Observed Bearing Angle

3.EXTENDED KALMAN FILTER

The state equation is linear, we need only determine

$$\mathrm{H}[n] = \left.\frac{\partial h}{\partial s[n]}\right|_{s[n]=\hat{s}[n|n-1]} \quad (16)$$

$$\frac{\partial h}{\partial s[n]} = \begin{bmatrix} \frac{r_x[n]}{R[n]} & \frac{r_y[n]}{R[n]} & 0 & 0 \\ \frac{-r_y[n]}{R[n]^2} & \frac{r_x[n]}{R[n]^2} & 0 & 0 \end{bmatrix} \quad (17)$$

We need to specify the covariance's of the driving noise and observation noise. If we assume that the wind gusts, speed corrections, etc is just as likely to occur in any directions and with the same magnitude, then it seems reasonable to assign the same variance to $u_x[n] and u_y[n]$ and to assume that they are independent with variance σ_u^2.The process noise covariance is specified as:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_u^2 & 0 \\ 0 & 0 & 0 & \sigma_u^2 \end{bmatrix} \quad (18)$$

In describing the variance of the measurement noise we note that the measurement error can be thought of as the estimation error of $\hat{R}[n]$ and $\hat{\beta}[n]$ .we usually assumes the estimation error to be zero mean. We usually assume the estimation errors to be independent and the variances to time invariant. Hence we have:

$$C = \begin{bmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_\beta^2 \end{bmatrix} \quad (19)$$

The extended Kalman filter equations for this problem are:

$$\hat{s}[n|n-1] = A\hat{s}[n-1|n-1] \quad (20)$$

$$M[n|n-1] = AM[n|n-1]A^T + Q \quad (21)$$

$$K[n] = M[n|n-1]H^T[n](C + H[n]M[n|n-1]H^T[n])^{-1} \quad (22)$$

$$\hat{s}[n|n] = \hat{s}[n|n-1] + K[n]\big(x[n] - h(\hat{s}[n|n-1])\big) \quad (23)$$

$$M[n|n] = (I - K[n]H[n])M[n|n-1] \quad (24)$$

Where

$$A = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (25)$$

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_u^2 & 0 \\ 0 & 0 & 0 & \sigma_u^2 \end{bmatrix} \quad (26)$$

$$C = \begin{bmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_\beta^2 \end{bmatrix} \quad (27)$$

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$$x[n] = \begin{bmatrix} \hat{R}[n] \\ \hat{\beta}[n] \end{bmatrix} \quad (28)$$

$$h(s[n]) = \begin{bmatrix} \sqrt{r_x^2[n] + r_y^2[n]} \\ \arctan \frac{r_y[n]}{r_x[n]} \end{bmatrix} \quad (29)$$

$$H[n] = \left. \frac{\partial h}{\partial s[n]} \right|_{s[n]=\hat{s}[n|n-1]} \quad (30)$$

$$H[n] = \left. \begin{bmatrix} \frac{r_x[n]}{R[n]} & \frac{r_y[n]}{R[n]} & 0 & 0 \\ \frac{-r_y[n]}{R[n]^2} & \frac{r_x[n]}{R[n]^2} & 0 & 0 \end{bmatrix} \right|_{s[n]=\hat{s}[n|n-1]} \quad (31)$$

4. INITIALIZATION

The extended Kalman filter is initialized as $\sigma_u^2 = 0.0001$, $\sigma_R^2 = 0.1$ and $\sigma_\beta^2 = 0.01$ where β is measured in radians. To employ an extended Kalman filter we must specify an initial state estimate .It is unlikely that we will have knowledge of the position and speed. Thus we choose an initial state that is quite far from the true one to check convergence propertyof Extendedversion of Kalman filter. In state equation we have assumed $\Delta = 1$

$$s[0] = \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

$$M[0] = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

5. RESULTS OF EXTENDED KALMAN FILTER

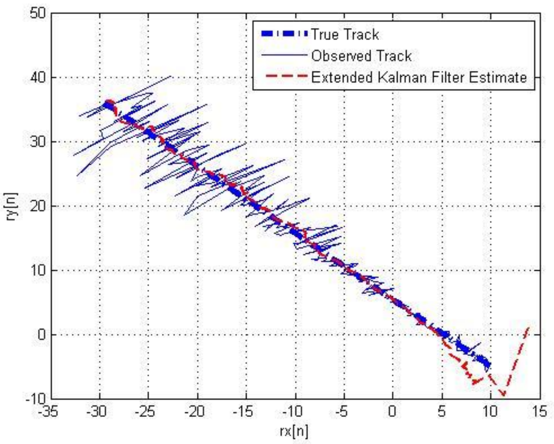

Fig. 4: Tracking of Trajectory using Extended Kalman Filter

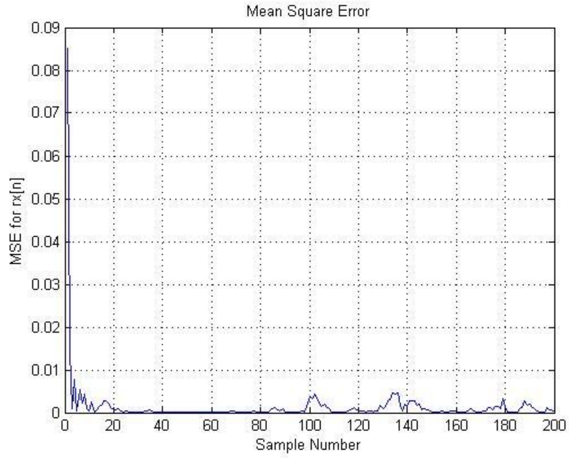

Fig. 5: Minimum MSE for $r_x[n]$

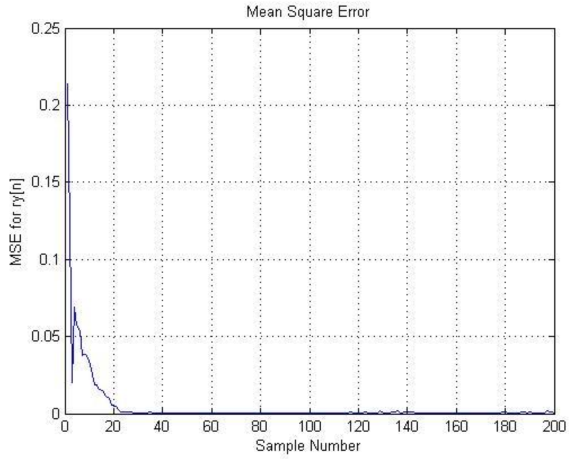

Fig. 5: Minimum MSE for $r_y[n]$

6. UNSCENTED KALMAN FILTER

Tracking of Vehicle is done using Unscented Kalman Filer and the results are compared with the Extended Kalman filter. The equations used for Unscented Kalman filter are described below.

Unscented Kalman Filter Parameters:

Weights for Mean Computation

$$\lambda = \alpha^2 M - M \quad (32)$$

- M is the dimension corresponding to state $x(n)^{MX1}$
- N is the dimension corresponding to the observation vector/matrix $y(n)^{NX1}$
- αa small +ve value that controls spread of sigma Points around mean of $x(n)$
- β represents prior Knowledge about $x(n)$

Weights for mean and Covariance:

$$w_0^m = \frac{\lambda}{M + \lambda} \quad (33)$$

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$$w_0^c = \frac{\lambda}{M+\lambda} + (1+\beta-\alpha^2) \quad (34)$$

$$w_i^m = w_i^c = \frac{0.5\,\lambda}{M+\lambda} \quad (35)$$

Qis the Co-variance matrix for Process Noise
C Co-variance matrix for Measurement Noise

Recursive Algorithm:

Choose Sigma Points

$$x_0(n-1) = \hat{x}(n-1) \quad (36)$$

$$x_i(n-1) = \begin{cases} \hat{x}(n-1) + \left(\sqrt{(M+\lambda)K(n-1)}\right)_i \; i = 1,2,-M \\ \hat{x}(n-1) - \left(\sqrt{(M+\lambda)K(n-1)}\right)_i \; i = M+1, M+2, -2M \end{cases} \quad (37)$$

Predict

$$\mathcal{X}_i(n-1) = A\hat{s}[n-1|n-1] \quad (38)$$

$$\hat{x}[n|n-1] = \sum_{i=0}^{2M} w_i^m\, \mathcal{X}_i(n-1) \quad (39)$$

$$K[n|n-1] = \sum_{i=0}^{2M} w_i^c\, (\mathcal{X}_i(n-1) - \hat{x}[n|n-1])(\mathcal{X}_i(n-1) - \hat{x}[n|n-1])^H + Q \quad (40)$$

$$Y_i[n|n-1] = \mathcal{C}(\mathcal{X}_i(n-1)) \quad (41)$$

$$\hat{y}[n|n-1] = \sum_{i=0}^{2M} w_i^m\, Y_i(n-1) \quad (42)$$

$$K_{yy}[n] = \sum_{i=0}^{2M} w_i^c\, (Y_i(n-1) - \hat{y}[n|n-1])(Y_i(n-1) - \hat{y}[n|n-1])^H + C \quad (43)$$

$$K_{xy}[n] = \sum_{i=0}^{2M} w_i^c\, (\mathcal{X}_i(n-1) - \hat{x}[n|n-1])(Y_i(n-1) - \hat{y}[n|n-1])^H \quad (44)$$

$$G[n] = K_{xy}[n]K_{yy}[n]^{-1} \quad (45)$$

$$K[n] = K[n|n-1] + G[n]K_{yy}[n]G[n]^H \quad (46)$$

$$\hat{x}(n) = \hat{x}(n|n-1) + G[n](y[n] - \hat{y}[n|n-1]) \quad (47)$$

7. INITIALIZATION

The Unscented Kalman filter is initialized as$\sigma_u^2 = 0.0001$, $\sigma_R^2 = 0.1$ and $\sigma_\beta^2 = 0.01$ where β is measured in radians. To employ an Unscented Kalman filter we must specify an initial state estimate .It is unlikely that we will have knowledge of the position and speed. Thus we choose an initial state that is quite far from the true one to check convergence of the Unscented Kalman filter. In state equation we have assumed$\Delta = 1$, $\alpha = 0.5$ and$\beta = 2$.

$$s[0] = \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

$$M[0] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8. RESULTS OF UNSCENTED KALMAN FILTER

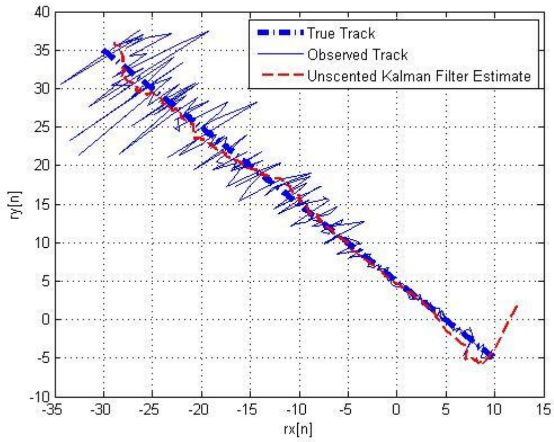

Fig. 6: Tracking of Trajectory using Unscented Kalman Filter

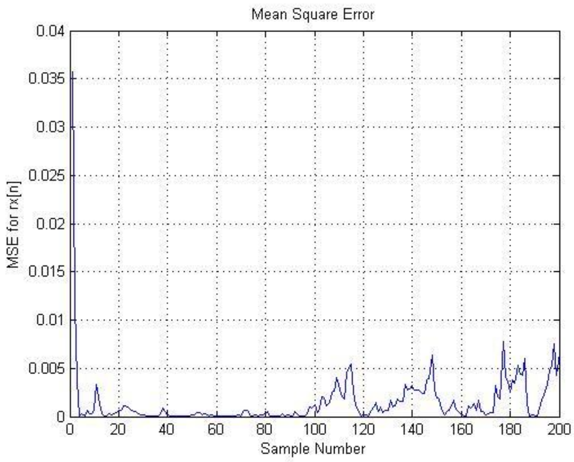

Fig. 7: Minimum MSE for $r_x[n]$

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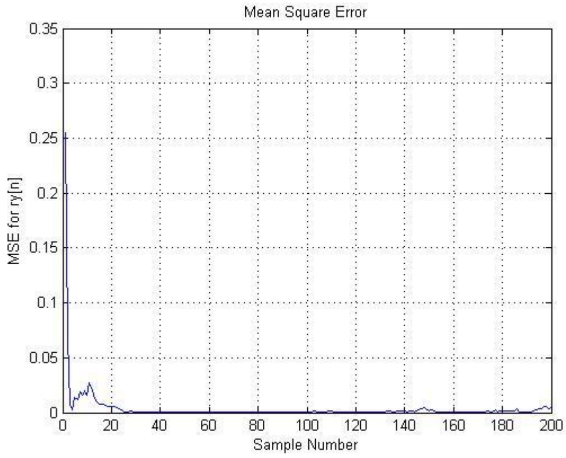

Fig. 8: Minimum MSE for $r_y[n]$

9.CONCLUSION

Tracking of Vehicle Trajectory is done using both Extended and Unscented Kalman Filter. Mean Square error of both filters were compared and it was analyzed that Unscented Kalman filter outperformed the Extended Kalman filter and it gives us better results in presence of strong nonlinearity in the state and observation equation.Secondly there is no need to compute Jacobian in Unscented Kalman filter. Unscented Kalman filter can be used in those scenarios where Jacobian is singular or Jacobian is hard to find.

10. ACKNOWLEDGEMENTS

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