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Chaotic Broadband Signals Control and Coupling using feedback Control Schemes

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ABSTRACT

Control and synchronization of the chaotic broadband carriers of two different chaotic systems is reported in this paper. The YU-WANG autonomous chaotic system is a three-dimensional system that possesses a quadratic cross-product and a hyperbolic nonlinear term in its algebraic structure. The Ai-Yuan-Zhi-Hao system consists of six control parameters in addition to three nonlinear terms. Firstly, linear feedback controllers are designed to regulate the trajectories of the state vectors of the two systems to asymptotically stable states based on the Routh-Hurwitz stability criteria. Secondly, hybrid feedback controllers are designed for identical and non-identical synchronization of the two systems. Numerical simulation of the models shows that the coupled systems are effectively controlled and synchronized by the designed controllers. The synchronized signals holds possibilities of applications in secure communication system for image encryption, signal masking and chaotic modulation.

Keywords: *Ai-Yuan-Zhi-Hao system, hybrid feedback control, hyperbolic nonlinearity, synchronization, YU-WANG system*

1. INTRODUCTION

In recent years, the number of research on chaotic phenomena has increased considerably due to the ever expanding frontiers of applications of chaos in engineering and non-engineering systems. Chaos is a phenomenon which results from the extreme sensitivity to perturbation in the structural parameters and initial conditions of some classes of dynamic systems. Chaotic signals have a random-like nature and broadband spectrum and are non-periodic [1]. Theoretically, a chaotic system can produce an infinite number of chaotic signals which are ergodic and non-periodic. As a result, with the continuous need for better security in communication systems, the broadband spectrum has been experimented upon and found to serve as effective spectral camouflage for message streaming through chaos-based communications systems while the high sensitivity feature can serve as an encryption key [2].
In secure communications systems, chaos can be used to mask signals or to serve as a modulator. In the chaos masking scheme, information is added directly onto the chaotic carrier, while preserving the algebraic structure producing the carrier. In chaos modulation however, the information is incorporated into the algebraic structure producing the carrier.

2. CHAOS SYNCHRONIZATION

Chaos synchronization occurs when two dissipative chaotic systems are coupled such that, in spite of the exponential divergence of their nearby trajectories, synchrony is achieved in their chaotic behaviours as $t \to \infty$. Since Pecora and Caroll's salient work [2], chaos synchronization has been widely studies under various perspectives. Several conditions influence synchronization of signals such as the type of synchronization scheme, parameter region of the systems and the degree of divergence of the two chaotic systems. The necessary condition for master-slave synchronization is that the non-driven slave subsystem must be asymptotically stable in the sense of Lyapunov [3]. Different methods have been developed to synchronize chaotic systems. These include adaptive control [4], active control [5], fuzzy control [6], and sliding mode control [7] among others, and recent years, these techniques have been used by various authors to synchronize different chaotic systems [8]- [15]. Chaos synchronization has been applied to multi-user time division multiplexing (TDM) communication systems [8]. A nonlinear control laws synchronize the message-masking trajectories of the two chaotic systems according to some conditions when the initial conditions of the master and slave systems are non-identical i.e. $x_1(0), x_2(0), x_3(0) \neq y_1(0), y_2(0), y_3(0)$. In chaos-based communication systems, the master and slave systems are respectively known as the transmitter and receivers.

A general block diagram demonstrating the principle of chaos synchronization for use in communication systems is shown in Fig. 1.

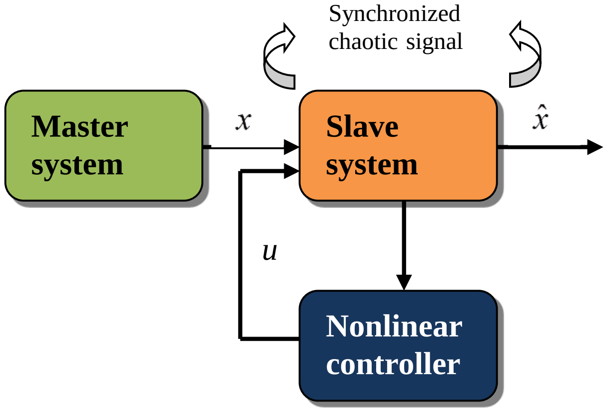

Fig.1. Principle of chaos synchronization [1]

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3. THE YU-WANG CHAOTIC SYSTEM

The Yu-Wang autonomous chaotic system [16] is a three-dimensional system that possesses a quadratic cross-product and a nonlinear term in the form of a hyperbolic sine or cosine function in its algebraic structure. Permutations in the algebraic structure can result to the evolution of a two-wing attractor, an anti-structure of the two-wing attractor and also a four-wing attractor with the same topology. Detailed structural and parametric analyses have been reported in [16]. Synchronization of the system via hybrid feedback control was reported in [17]. In this section, attention is focused on the regulation of the system state trajectories to some equilibrium points at the origin using feedback controller technique. The governing equation of the system can be expressed as:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \beta x_1 - \gamma x_1 x_3 \\ \dot{x}_3 &= f(t) - \eta x_3 \end{aligned} \quad (1)$$

Where x_1, x_2, x_3 are state variables, $\alpha, \beta, \gamma, \eta > 0$ are positive constants and $f(t)$ is a changeable nonlinear hyperbolic function of the form $\sinh(x_1 x_2)$ or $\cosh(x_1 x_2)$. For values of $\alpha = 10, \beta = 30, \gamma = 2, \eta = 2.5$ and the $\sinh(x_1 x_2)$ hyperbolic function, the system evolves the attractor and time series trajectories shown in Fig. 2 and Fig.3 respectively.

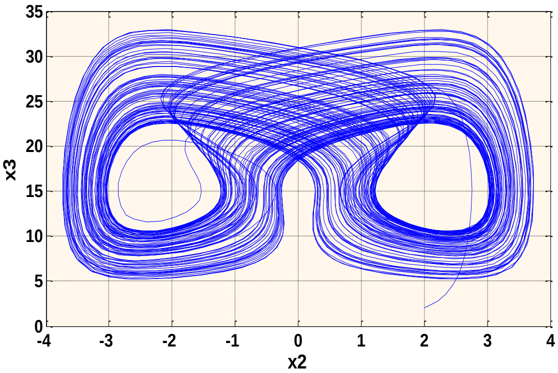
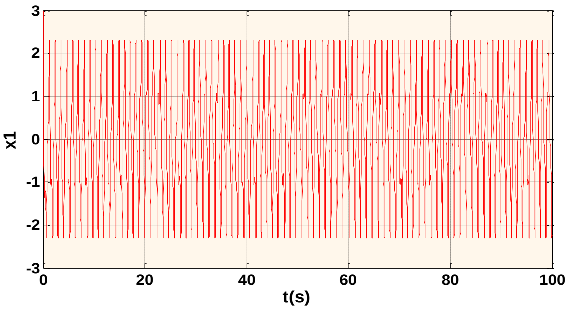
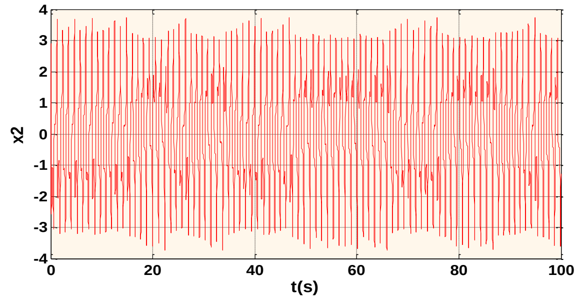


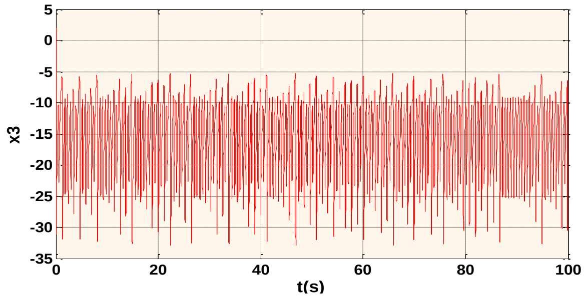
Fig.2. Chaotic attractor in y-z plane



(a)



(b)



(c)

Fig.3. State trajectories of the Yu-Wang system

4. THE AI-YUAN-ZHI-HAO CHAOTIC SYSTEM

The Ai-Yuan-Zhi-Hao chaotic system is a three-dimensional system that exhibits complex dynamics for selected values of its control parameters [18]. It consists of six control parameters and three nonlinear terms in its algebraic structure and is represented by the following three coupled differential equations

$$\begin{aligned} \dot{y}_1 &= b(y_2 - y_1) + c y_2 y_3 \\ \dot{y}_2 &= d y_1 - m y_1 y_3 \\ \dot{y}_3 &= -n y_3 + p y_1^2 \end{aligned} \quad (2)$$

Where y_1, y_2, y_3 are state variables, $b, c, d, m, n, p > 0$ are positive constants. For values of $b = 10, c = 25, d = 20, m = 25, n = 7, p = 3$, system (2) exhibits chaotic behaviours as shown in Fig. 4 and Fig.5..

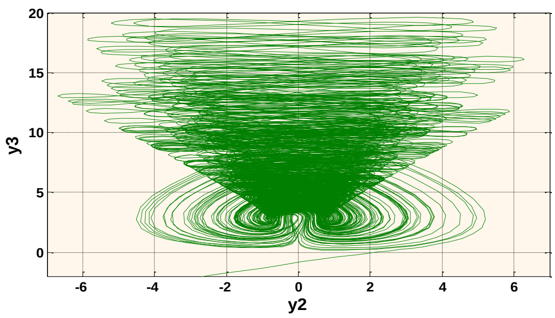


Fig. 4. Chaotic attractor in y-z plane

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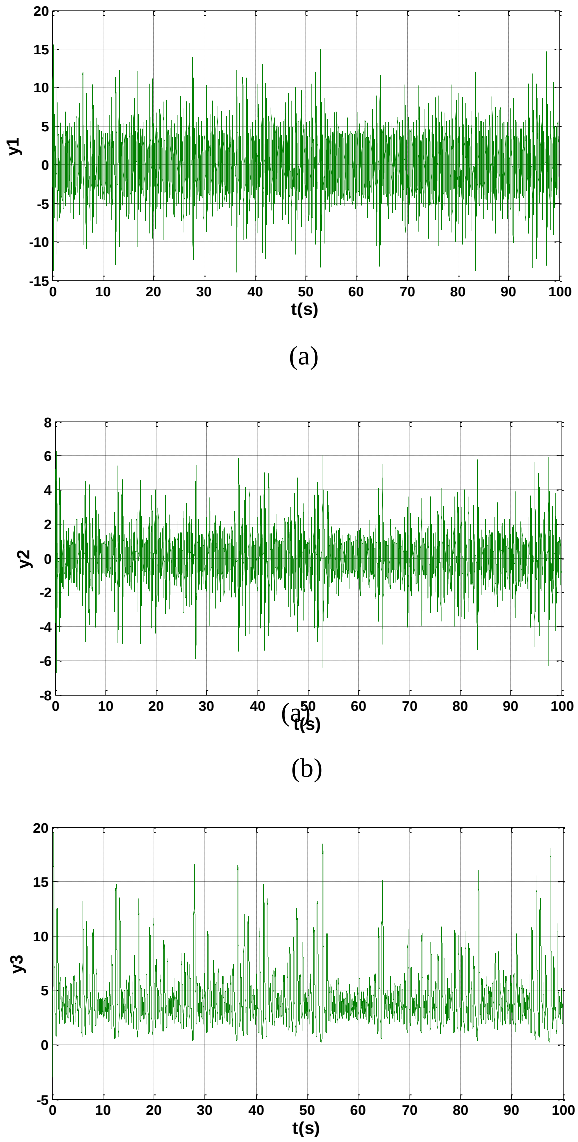

(a)

(b)

Fig.5. State trajectories of the Ai-Yuan-Zhi-Hao system

Stabilization of the Ai-Yuan-Zhi-Hao system via fuzzy controllers has been reported in [19].

5. LINEAR FEEDBACK CONTROLLER DESIGN

(a) Control of the Yu-Wang system

Feedback control principle is a classical control strategy that has been extensively used in the control of dynamic plants. In recent years, there had been some improved versions of feedback control techniques such as multivariable feedback control [20] among others. In this section, we summarize the design of a linear feedback controller [21]. In order to stabilize the chaotic system via feedback controllers, the system's algebraic structure (1) is first linearized at some equilibrium point $E(0,0,0)$ to produce the Jacobian matrix of the following form:

$$J_{Yu}\big|_{0,0,0} = \begin{bmatrix} -\alpha & \alpha & 0 \\ \beta & 0 & 0 \\ 0 & 0 & -\eta \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 \\ 30 & 0 & 0 \\ 0 & 0 & -2.5 \end{bmatrix}$$

The matrix (2) results in the polynomial

$$\lambda^3 + 12.5\lambda^2 - 275\lambda - 750 \qquad (4)$$

The roots of (3) are

$$\begin{aligned} \lambda_1 &= -23.0278 \\ \lambda_2 &= 13.0278 \\ \lambda_3 &= -2.5000 \end{aligned} \qquad (5)$$

The equilibrium point is a saddle-focus point and the system is therefore unstable at this equilibrium point. Let the controllers to be designed be represented as

$$\begin{bmatrix} \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{bmatrix} = -\Delta \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (6)$$

Where $\varpi = [\varpi_{11}, \varpi_{22}, \varpi_{33}]^T$ the feedback controller subsystems are associated with each state vector and Δ is a diagonal matrix whose diagonal elements $diag[\xi_{11}, \xi_{22}, \xi_{33}]$ are the gain coefficients of the feedback controllers.

By using (6) in (1), the controlled Yu-Wang system becomes

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) - \xi_{11}x_1 \\ \dot{x}_2 &= \beta x_1 - \gamma x_1 x_3 - \xi_{22}x_2 \\ \dot{x}_3 &= f(t) - \eta x_3 - \xi_{33}x_3 \end{aligned} \qquad (7)$$

The characteristic equation of (7) reduces to

$$\lambda^3 + A\lambda^2 + B\lambda + C \qquad (8)$$

Where

$$\begin{aligned} A &= \alpha + \eta + \xi_{11} + \xi_{22} + \xi_{33} \\ B &= \alpha\eta - \alpha\beta + \alpha\xi_{22} + \alpha\xi_{33} + \eta\xi_{11} + \ldots \\ &\quad + \eta\xi_{22} + \xi_{11}\xi_{22} + \xi_{11}\xi_{33} + \xi_{22}\xi_{33} \\ C &= \alpha\beta\eta + \alpha\eta\xi_{22} - \alpha\beta\xi_{33} + \eta\xi_{11}\xi_{22} + \ldots \\ &\quad + \alpha\xi_{22}\xi_{33} + \xi_{11}\xi_{22}\xi_{33} \end{aligned} \qquad (9)$$

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According to the Routh Hurwitz stability criteria [22], the system (1) is asymptotically stable if and only if the roots of (7) are negative. In order to satisfy these criteria, (9) must satisfy the following conditions

$$\begin{aligned} &A > 0 \\ &B > 0 \\ &AB > C \end{aligned} \tag{10}$$

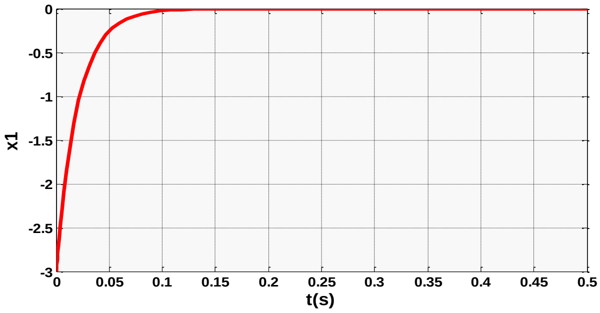
The feedback gain coefficients are appropriately chosen such that (10) is satisfied and the system is asymptotically stable.

6. NUMERICAL SIMULATION RESULTS

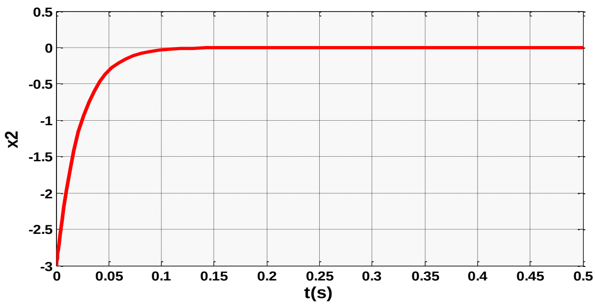
The controlled Yu-Wang system (7) was simulated for the following initial conditions

$x|_{x_1(0),x_2(0),x_3(0)} = [-3,-3,-3]$ and feedback gains

$[\xi_{11}, \xi_{22}, \xi_{33}] = [50, 70, 50]$



(a)



(b)

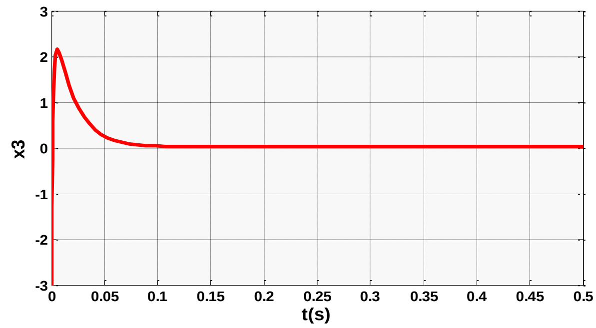


Fig. 6. Controlled state trajectories of the Yu-Wang system

(b) Control of the Ai-Yuan-Zhi-Hao system

At the equilibrium point $E(0,0,0)$, the Jacobian matrix of (2) is given by

$$J_{Ai}|_{0,0,0} = \begin{bmatrix} -b & b & 0 \\ d & 0 & 0 \\ 0 & 0 & -n \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 \\ 20 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix} \tag{11}$$

The matrix (11) has the polynomial

$$\lambda^3 + 13\lambda^2 - 170\lambda - 600 \tag{12}$$

Whose roots are computed as

$$\begin{aligned} \lambda_1 &= -20.00 \\ \lambda_2 &= 10.00 \\ \lambda_3 &= -3.00 \end{aligned} \tag{13}$$

The equilibrium is also a saddle-focus point. The system is therefore unstable at this equilibrium point.

Using similar relationship as in (6), the controlled Ai-Yuan-Zhi-Hao system becomes

$$\begin{aligned} \dot{y}_1 &= b(y_2 - y_1) + cy_2y_3 - \psi_{11}y_1 \\ \dot{y}_2 &= dy_1 - my_1y_3 - \psi_{22}y_2 \\ \dot{y}_3 &= -ny_3 + py_1^2 - \psi_{33}y_3 \end{aligned} \tag{14}$$

The characteristic equation of (14) is

$$\lambda^3 + A\lambda^2 + B\lambda + C \tag{15}$$

Where

$$\begin{aligned} A &= b + n + \psi_{11} + \psi_{22} + \psi_{33} \\ B &= bn - bd + b\psi_{22} + b\psi_{33} + n\psi_{11} + \ldots \\ &\quad + n\psi_{22} + \psi_{11}\psi_{22} + \psi_{11}\psi_{33} + \psi_{22}\psi_{33} \\ C &= bdn + bn\psi_{22} - bd\psi_{33} + n\psi_{11}\psi_{22} + \ldots \\ &\quad + b\psi_{22}\psi_{33} + \psi_{11}\psi_{22}\psi_{33} \end{aligned} \tag{16}$$

For stabilization of the state trajectories, the Hurwitz conditions (10) must be satisfied.

7. NUMERICAL SIMULATION RESULTS

The controlled Ai-Yuan-Zhi-Hao system (14) was simulated for the following initial conditions

$y|_{y_1(0),y_2(0),y_3(0)} = [2,1,0]$ and feedback gains

$[\psi_{11}, \psi_{22}, \psi_{33}] = [50, 80, 50]$

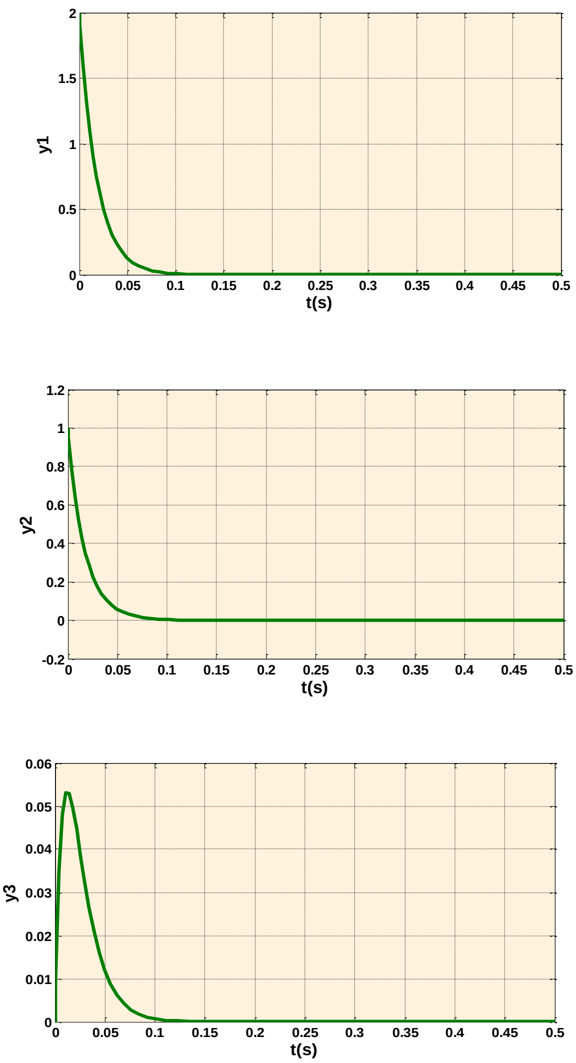


Fig.7. Controlled state trajectories of the Ai-Yuan-Zhi-Hao

7. HYBRID FEEDBACK CONTROLLER DESIGN

Hybrid feedback control provides a convenient approach to combined linear and nonlinear controller subsystems into a single controller to globally regulate a system [23].

Consider a master chaotic system modelled in the form of differential equation of the form

$$\dot{x} = Gx + H\tau(x) \tag{17}$$

Where $x \in R^n$ is the state vector, $G \in R^{n\times n}$ is the system matrix, $H \in R^{n\times n}$ is a constant matrix and $\tau(x): R^n \to R^n$ is a nonlinear vector function. Let the controlled slave system which is to be synchronized with the master system be expressed as

$$\dot{y} = Jy + H[\tau(y) + u] \tag{18}$$

u is a hybrid feedback controller to be designed and is composed of

$$u = u_{NL} + u_L \tag{19}$$

where u_{NL} and u_L are the nonlinear and linear controller subsystems of the global controller respectively and Where Ξ is the feedback gains, $e = (e_1, e_2, e_3)^T$ is the synchronization errors where $x = (x_1, x_2, x_3)^T, y = (y_1, y_2, y_3)^T$ and

$$\begin{aligned} e_1 &= x_1 - y_1 \\ e_2 &= x_2 - y_2 \\ e_3 &= x_3 - y_3 \end{aligned} \tag{20}$$

In identical synchronization, the system matrices of the master and slave are identical i.e $G = J$

Let $\Xi = diag[\Xi_1, \Xi_2, \Xi_3]$ where Ξ_1, Ξ_2, Ξ_3 are the feedback gains associated with the linear controller subsystem. In other to satisfy the necessary and sufficient conditions for synchronization, the error system is modelled according to the following structure [24]

$$e' = (G - H\Xi)e \tag{21}$$

Thus, by solving the matrix $G - H\Xi$, we have

$$\begin{aligned} u_{NL} &= \tau(x) - \tau(y) \\ u_L &= \Xi(x - y) \end{aligned} \tag{22}$$

According to Routh Hurwitz stability criterion, if the matrix $G - H\Xi$ has all its eigenvalues in the negative real parts, then the sufficient condition for synchronization is satisfied and the error system (21) will be an asymptotically stable and the two systems (17) and (18) would be synchronized as $t \to \infty$.

Case 1: Identical synchronization of the Yu-Wang system

Let the master system (1) be represented in the form of (17) as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\alpha & \alpha & 0 \\ \beta & 0 & 0 \\ 0 & 0 & -\eta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ x_1 x_3 \\ \sinh(x_1 x_2) \end{bmatrix} \tag{23}$$

By comparing (23) with (17), it can be seen that

$$G=\begin{bmatrix}-\alpha & \alpha & 0\\ \beta & 0 & 0\\ 0 & 0 & -\eta\end{bmatrix}; H=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix};$$
$$\tau(x)=\begin{bmatrix}0\\ x_1x_3\\ \sinh(x_1x_2)\end{bmatrix} \quad (24)$$

Let the feedback gain matrix be given by

$$\Xi=\begin{bmatrix}\Xi_1 & 0 & 0\\ 0 & \Xi_2 & 0\\ 0 & 0 & \Xi_3\end{bmatrix} \quad (25)$$

Then

$$G-H\Xi=\begin{bmatrix}-\alpha-\Xi_1 & a & 0\\ \beta & -\Xi_2 & 0\\ 0 & 0 & -\eta-\Xi_3\end{bmatrix} \quad (26)$$

The characteristic equation of (13) is given by

$$\begin{bmatrix}\lambda+\alpha+\Xi_1 & a & 0\\ \beta & \lambda+\Xi_2 & 0\\ 0 & 0 & \lambda+\eta+\Xi_3\end{bmatrix} \quad (27)$$

$$=\lambda^3+\delta\lambda^2+\varepsilon\lambda+\Phi \quad (28)$$

The roots of the polynomial (28) will satisfy the Hurwitz condition for synchronization if $\delta>0$, $\varepsilon>0$ and $\delta\varepsilon>\Phi$. These conditions will be satisfied when $\Xi_1,\Xi_2,\Xi_3>0$ are appropriately chosen to drive the error system into an asymptotically stable state.
By the using the following notations for the state vectors of the slave system x_4, x_5, x_6, the hybrid feedback controllers can be then be written in the following forms:

$$u_{NL}=\begin{bmatrix}0\\ x_1x_3+x_4x_6\\ \sinh(x_1x_2)-\sinh(x_4x_5)\end{bmatrix}$$
$$u_L=\begin{bmatrix}\Xi_1(x_1-x_4)\\ \Xi_2(x_2-x_5)\\ \Xi_3(x_3-x_6)\end{bmatrix} \quad (29)$$

By using (24) in conjunction with (29), the controlled slave system (3) can be written in the following form:

$$\begin{aligned}\dot{x}_4&=\alpha(x_5-x_4)-\Xi_1(x_1-x_4)\\ \dot{x}_5&=\beta x_4-\gamma x_1x_3-\Xi_2(x_2-x_5)\\ \dot{x}_6&=\sinh(x_1x_2)-\eta x_6-\Xi_3(x_3-x_6)\end{aligned} \quad (30)$$

7. NUMERICAL SIMULATION RESULTS

The master (17) and the controlled slave (30) systems were simulated with MATLAB software for the following initial conditions: Master system, $x_m\big|_{x_1(0),x_2(0),x_3(0)}=[1,-9,8]$ and the Slave system, $x_s\big|_{x_4(0),x_5(0),x_6(0)}=[6,3,1]$ and the feedback gains $\Xi_1=\Xi_2=\Xi_3=100$. The following plots depict the simulation results.

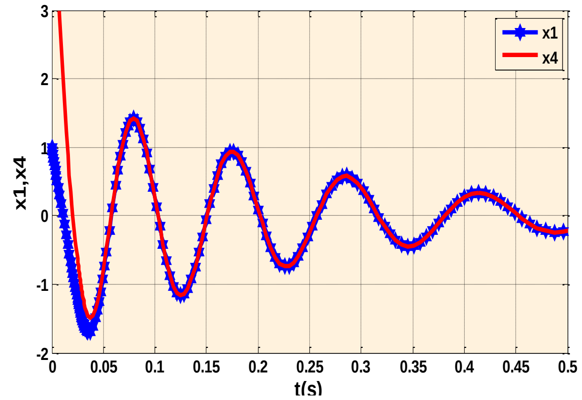


Fig.8. Synchronized trajectories of state vectors x1, x4

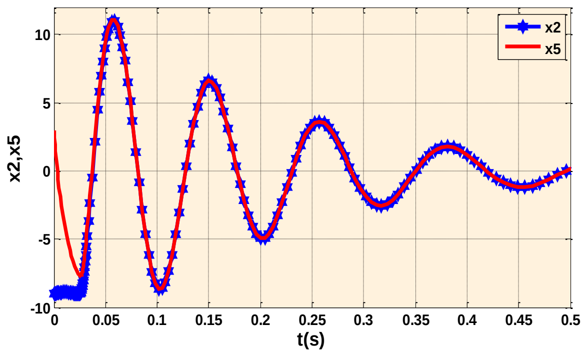


Fig.9. Synchronized trajectories of state vectors x2, x5

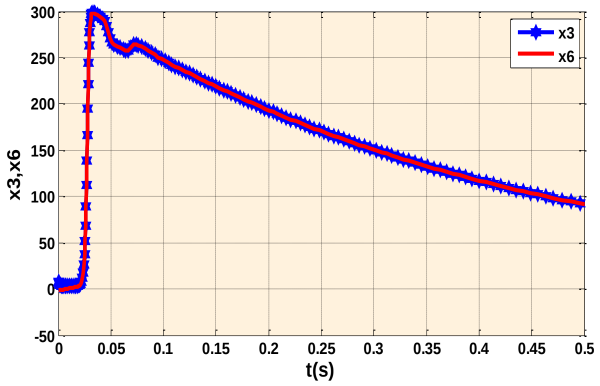


Fig.10. Synchronized trajectories of state vectors x3, x6

Case 2: Identical synchronization of the Ai-Yuan-Zhi-Hao system

Let the master system (2) be represented in the form of (17) as follows

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} -b & b & 0 \\ d & 0 & 0 \\ 0 & 0 & -n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_2 y_3 \\ y_1 y_3 \\ y_1^2 \end{bmatrix} \tag{31}$$

By comparing (31) with (17), it can be deducted that

$$G = \begin{bmatrix} -b & b & 0 \\ d & 0 & 0 \\ 0 & 0 & -n \end{bmatrix}; H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \tau(x) = \begin{bmatrix} y_2 y_3 \\ y_1 y_3 \\ y^2 \end{bmatrix} \tag{32}$$

By employing the same reasoning in (25), (26), we have the feedback gain matrix

$$\Xi = \begin{bmatrix} \Xi_1 & 0 & 0 \\ 0 & \Xi_2 & 0 \\ 0 & 0 & \Xi_3 \end{bmatrix} \tag{33}$$

Then

$$G - H\Xi = \begin{bmatrix} -b-\Xi_1 & b & 0 \\ d & -\Xi_2 & 0 \\ 0 & 0 & -n-\Xi_3 \end{bmatrix} \tag{34}$$

The characteristic equation of (13) is given by

$$\begin{bmatrix} \lambda + b + \Xi_1 & b & 0 \\ d & \lambda + \Xi_2 & 0 \\ 0 & 0 & \lambda + n + \Xi_3 \end{bmatrix} \tag{35}$$

$$= \lambda^3 + \delta\lambda^2 + \varepsilon\lambda + \Phi \tag{36}$$

To satisfy the Hurwitz constraints, the roots of (36) must be $\delta > 0$, $\varepsilon > 0$ and $\delta\varepsilon > \Phi$ and the two identical system will synchronized in finite time. Using the following notations for the slave system state vectors, y_4, y_5, y_6, the hybrid feedback controller is thus constructed as follows

$$u_{NL} = \begin{bmatrix} y_2 y_3 - y_5 y_6 \\ y_1 y_3 - y_4 y_6 \\ y_1^2 - y_4^2 \end{bmatrix}, \quad u_L = \begin{bmatrix} \Xi_1 (y_1 - y_4) \\ \Xi_2 (y_2 - y_5) \\ \Xi_3 (y_3 - y_6) \end{bmatrix} \tag{37}$$

The controlled slave system transforms into the three-coupled equation given by

$$\begin{aligned} \dot{y}_4 &= b(y_5 - y_4) + c y_2 y_3 - \Xi_1 (y_1 - y_4) \\ \dot{y}_5 &= d y_4 - m y_1 y_3 - \Xi_2 (y_2 - y_5) \\ \dot{y}_6 &= -n y_6 + p y_4^2 - \Xi_3 (y_3 - y_6) \end{aligned} \tag{38}$$

7. NUMERICAL SIMULATION RESULTS

The master (17) and controlled slave (37) systems were simulated with MATLAB software for the following initial conditions: Master system, $y_m \big|_{y_1(0), y_2(0), y_3(0)} = [1, -9, 8]$ and the Slave system, $x_s \big|_{x_4(0), x_5(0), x_6(0)} = [6, 3, 1]$ and the feedback gains $\Xi_1 = \Xi_2 = \Xi_3 = 100$. The following plots depict the simulation results.

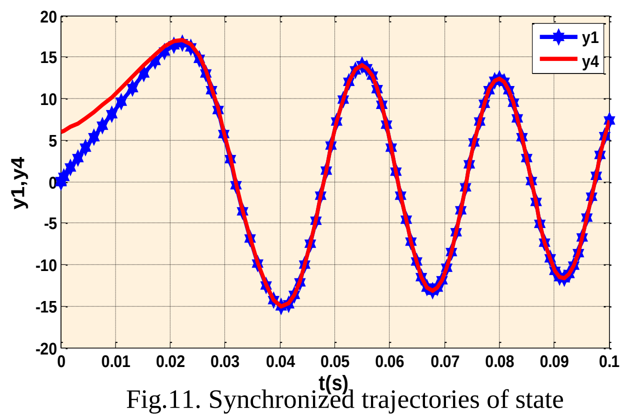


Fig.11. Synchronized trajectories of state vectors y1, y4

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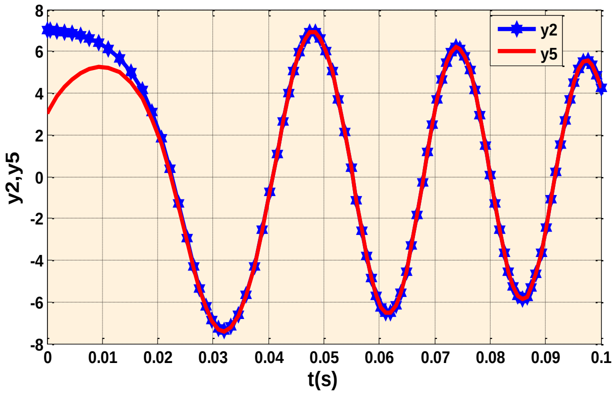

Fig.12. Synchronized trajectories of state vectors y2, y5

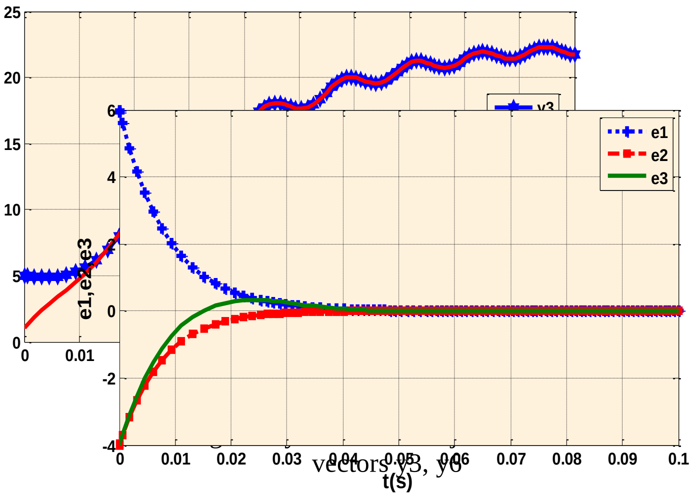

Fig.14. Converged trajectories of the error state vectors e1, e2, e3

Case 3: Synchronization between Yu-Wang and Ai-Yuan-Zhi-Hao systems

In this section, the two diffeomorphic systems are synchronized via hybrid feedback controllers. Let (1) be the master system and (2) be the slave system. Let the master system be represented as in (23)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-\alpha & \alpha & 0\\ \beta & 0 & 0\\ 0 & 0 & -\eta\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\ x_1x_3\\ \sinh(x_1x_2)\end{bmatrix} \quad (39)$$

And the slave as

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{bmatrix}=\begin{bmatrix}-b & b & 0\\ d & 0 & 0\\ 0 & 0 & -n\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}y_2y_3\\ y_1y_3\\ y_1^{\,2}\end{bmatrix} \quad (40)$$

Based on the relationship (17) and (18), It can be seen from (38) and (39) that $G \neq J$, hence, the hybrid feedback controller is computed as follows

$$u_{NL}=\begin{bmatrix}-y_2y_3\\ x_1x_3-y_1y_3\\ \sinh(x_1x_2)-y_1^{\,2}\end{bmatrix}$$
$$u_L=\begin{bmatrix}\Xi_1(x_1-y_1)\\ \Xi_2(x_2-y_2)\\ \Xi_3(x_3-y_3)\end{bmatrix} \quad (41)$$

By using (18), the controlled slave system is

$$\begin{aligned}\dot{y}_1&=b(y_2-y_1)-\Xi_1(x_1-y_1)\\ \dot{y}_2&=dy_4-mx_1x_3-\Xi_2(x_2-y_2)\\ \dot{y}_3&=\sinh(x_1x_2)-ny_3-\Xi_3(x_3-y_3)\end{aligned} \quad (42)$$

7. NUMERICAL SIMULATION RESULTS

The master (17) and controlled slave (37) systems were simulated with MATLAB software for the following initial conditions: Master system $x|_{x_1(0),x_2(0),x_3(0)}=[6,3,8]$ and the and the Slave system $y|_{y_1(0),y_2(0),y_3(0)}=[1,3,1]$. The following plots depict the simulation results.

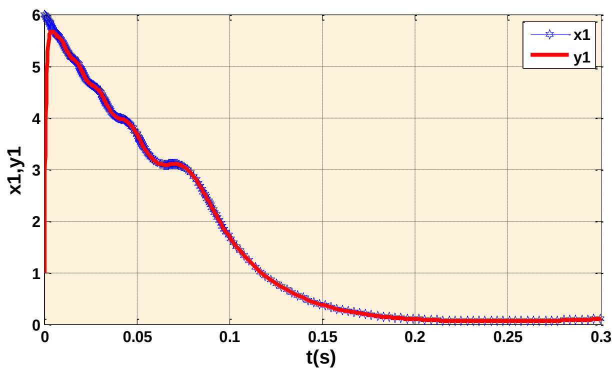

Fig.15. Synchronized trajectories of state vectors x1, y1

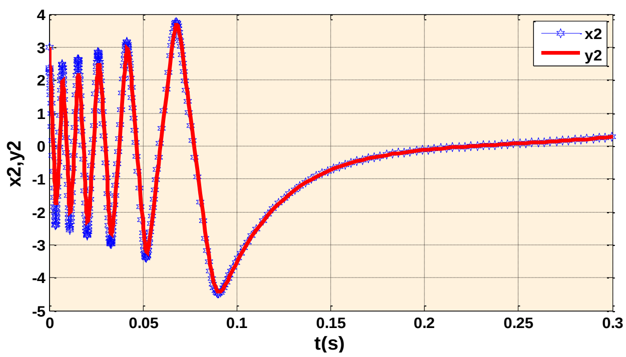

Fig.16. Synchronized trajectories of state vectors x2, y2

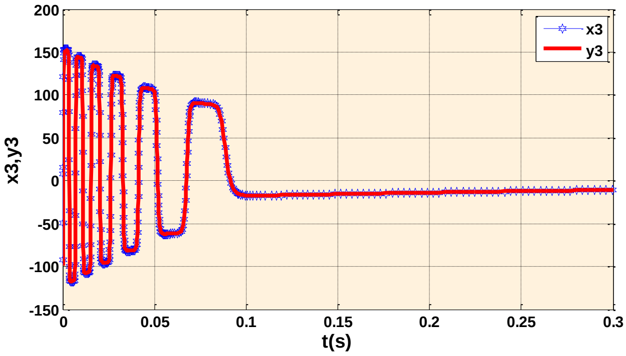

Fig.17. Synchronized trajectories of state vectors x3, y3

8. CONCLUSION

Linear and hybrid feedback control-based regulation and synchronization of the chaotic broadband carriers of two diffeomorphic chaotic systems- Yu-Wang and Ai-Yuan-Zhi-Hao systems is reported in this paper. The system matrices of both systems were found to be identical i.e. diffeomorphic but possesses dissimilar topological characteristics. The synchronization scheme was robust even when different hyperbolic functions such as *sinh*, *cosh* and *tanh* were arbitrarily introduced into the controlled slave subsystems. However, both systems are extremely sensitive to parametric perturbations as simulations shown stiffness in evolution of attractors when hyperbolic functions other than the *sinh* and *cosh* are injected into the deactivated controller architecture of the Yu-Wang system. The synchronized trajectories of the two systems were too closed for a time series that is greater than 0.1(s).

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