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PAPR Reduction in OFDM using Non-Linear Companding Scheme

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ABSTRACT

Orthogonal frequency division Multiplexing (OFDM) is a new multicarrier modulation scheme and strongly efficient in bandwidth usage. It have less ICI (Inter carrier interference), ISI(Inter symbol interference) and good spectral and power efficiency. OFDM provide a high speed wireless communication and most high speed wireless communication system uses OFDM for transmission e.g. IEEE 802.11, IEEE 802.16, IEEE 802.20..An OFDM baseband signal is the sum of a number of orthogonal sub-carriers. The sub-carriers being independently modulated by using QAM or PSK or by its own data. The orthogonality between subcarrier allows a lot of sub-carriers can be transmitted simultaneously in a tight frequency space without interference from each other by saving the bandwidth and they are able to overlap without interfering. So OFDM systems provide good spectral efficiency without causing adjacent channel interference (ACI). Since many subcarrier components are added via an inverse fast Fourier transformation (IFFT) operation in the time domain so the transmit signals in an orthogonal frequency-division multiplexing (OFDM) system can have high peak values. As a result, OFDM systems as compare to single carrier system are known to have a high peak-to-average power ratio (PAPR). PAPR is a major problem, this paper presents a hybrid PAPR reduction technique, which uses both clipping and non-linear companding technique. The original signal is transformed using the proposed hybrid technique, and again using reverse companding scheme, original signal can be recovered error free. To prove hypothesis simulation results are also presented. .

Keywords: OFDM, PAPR, ICI, ISI, Non-linear companding.

1. INTRODUCTION

Wireless Communication Systems have a greater demand for increased data rates due to the new emerging technologies. The bandwidth of the service provider is very limited. The wireless channel is statistical in nature and the error rates are poorer in such types of communication system than that of its counterpart. However, these new technologies are hindered by various factors such as ISI and fading due to many reasons. OFDM has been proposed to contend with all these types of hindrances. As the wireless channel estimation is statistical in nature so for OFDM should capture both the time and frequency domain characteristics. This has been accomplished by taking the product of the correlation function in time and frequency domains. Such a criterion is used to minimize the variance of estimation errors disparate to the earlier estimation types which uses a priori knowledge of the channel estimation [1-4]. Efficient use of radio spectrum can be achieved by placing modulated carriers as close as possible without causing Inter-Carrier Interference (ICI). Optimally, the bandwidth of each carrier would be adjacent to its neighbours, so there would be no wasted spectrum. In practice, a guard band must be placed between each carrier bandwidth to provide a space where a filter can attenuate an adjacent carrier's signal. These guard bands are wasted bandwidth. In order to transmit high data rates, short symbol periods must be used. The symbol period is the inverse of the baseband data rate ($T = 1/R$), so as R increases, T must decrease. Inter-Symbol Interference (ISI) is also depend on symbol period if symbol period is small then there is large probability of ISI in a multi-path environment. Orthogonal Frequency Division Multiplexing (OFDM) addresses both of these problems. OFDM gives a technique that allowing the bandwidths of modulated carriers to overlap without interference (no ICI). It also provides a high date rate with long symbol duration, thus helping to eliminate ISI.OFDM may therefore be considered as a candidate modulation technique in a broadband, multi-path environment.

Since many subcarrier components are added via an inverse fast Fourier transformation (IFFT) operation in the time domain so the transmit signals in an orthogonal frequency-division multiplexing (OFDM) system can have high peak values. As a result, OFDM systems as compare to single carrier system are known to have a high peak-to-average power ratio (PAPR). The efficiency of power amplifier of transmitter and signal-to-quantization noise ratio (SQNR) depend on PAPR As soon as PAPR increases the SQNR and efficiency of power amplifier is decreases.

The PAPR of a signal is given by the following formula:

$$PAPR_{dB} = 10\log\left(\frac{\max[x(n)x^{*}(n)]}{E[x(n)x^{*}(n)]}\right) \quad (1)$$

Where ()* corresponds to the conjugate operator.

2. RELATED WORK

For PAPR reduction, various mechanism is proposed in past, and are defined as Clipping and Filtering [5], Selective Mapping (SLM) [7], Block Coding [10], Partial Transmit Sequence (PTS) [11-19], Interleaving, Tone Reservation (TR) [20], Tone Injection (TI) [20]. All of the mentioned technique uses some kind of mechanism to reduce PAPR. However, in recent methods non-linear companding schemes are preferred over other methods [11]. Two of the notable schemes are as under:

Peak-to-Average Power Ratio Reduction of OFDM Signals with Nonlinear Companding Scheme [21]

In this paper we proposed a novel nonlinear companding scheme which reduces the PAPR and gives better Bit Error Rate (BER) for OFDM systems. In this scheme we compressing the large signal and constant average power maintained by selecting proper transform parameter. In analysis we find that this scheme can also offer a good BER performance at the receiver without de-companding. The final result of simulation shows in terms of PAPR reduction, BER performance and in terms of spectrum side-lobe gives better performance than other companding scheme. The obtained PAPR is just above 4 dB and for BER=10^{-3} the required E_b/N_0 is nearly 7 dB.

A Piecewise Linear Companding Transform for PAPR Reduction of OFDM Signals with Companding Distortion Mitigation [22]

This paper proposed a new piece-wise linear companding scheme and its main aim to reduce companding distortion. The theoretical characterization of companding distortion is study in the design of companding transform. It describes for reducing companding distortion we should do companding of larger signals with smaller amplitude increments are more effective. On the basis of analysis of results, for a signal if amplitude is greater than the companded peak amplitude then we do clipping for peak power reduction and do the linear transformation of signal if the amplitude of signal is close to companded peak amplitude for power compensation.. With the careful design of the companded peak amplitude and the linear transform scale, the proposed scheme can achieve better BER and power spectral density (PSD) performance, while reducing PAPR effectively by careful design of the companded peak amplitude.

3. PROPOSED METHOD

In this section a novel hybrid companding scheme is proposed. The proposed companding scheme is nonlinear is nature. The proposed function is a piece-wise linear in nature. The main aim of the scheme is to reduce the amplitude variations and thus reducing PAPR. Now to reduce variations in amplitudes two methods can be followed

1. As a first transformation, lower amplitudes can be amplified while keeping higher amplitude values to fixed levels. But in this case, lower values can be saturated and during inverse process, recovery of original amplitudes may not be possible.
2. In the second method, lower amplitude levels are kept as it is, but the higher values of amplitude are compressed, thus variation reduces. This technique is more realistic and feasible.

When the initial discrete signal x_n is companded with a given peak amplitude *Ac*, the proposed companding scheme shown in Fig.1 clips the signals with amplitudes more than Ac for peak power reduction, and linearly transforms the signals with amplitudes which is more than *Ai* and close to *Ac* for power compensation. Then, the companding function of the proposed companding scheme can be formulated as

$$h(x) = \begin{cases} x & |x| \le A_i \\ kx + (1-k)A_c & A_i \le |x| \le A_c \\ \operatorname{sgn}(x) A_c & |x| > A_c \end{cases} \qquad (2)$$

The above equation for positive value of *x* can be written as

$$h(x) = \begin{cases} x & x \le A_i \\ kx + (1-k)A_c & A_i \le x \le A_c \\ A_c & x > A_c \end{cases} \qquad (3)$$

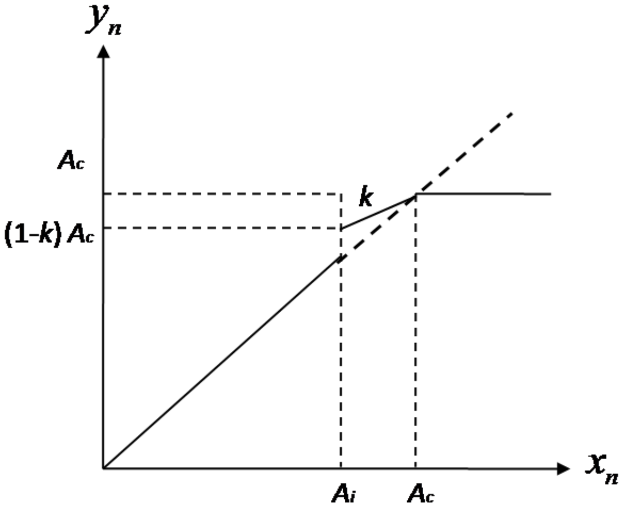

Figure 1: Proposed Companding Function (Pictorial Representation)

As the transform will be reversed at the receiver, therefore it inverse should exist. For inverse to exist, the function should be one-to-one. First checking one-to-one correspondence, let $y_1 = kx_1 + (1-k)A_c$ then

For one-to-one if $y_1 = y_2$ then $x_1 = x_2$.

$kx_1 + (1-k)A_c = kx_2 + (1-k)A_c \Rightarrow x_1 = x_2$.

Now evaluating inverse function

$$y = kx + (1-k)A_c \Rightarrow x = \frac{y-(1-k)A_c}{k}$$

Consequently, the decompounding function at the receiver is

$$h^{-1}(x) = \begin{cases} x & |x| \le A_i \\ \dfrac{x-(1-k)A_c}{k} & (1\text{-}k)A_c \le |x| \le A_c \\ \operatorname{sgn}(x) A_c & |x| > A_c \end{cases} \qquad (4)$$

Corresponding CDF is

$$H(x) = \begin{cases} \dfrac{x^2}{2} & x \le A_i \\ k\dfrac{x^2}{2} + (1-k)A_c x & A_i \le x \le A_c \\ A_c & x > A_c \end{cases} \qquad (5)$$

It is obvious that the proposed companding transform is specified by parameters A_c , A_i and k. With the premise of keeping the typical signal power constant, k has to be a positive real number smaller than one. Besides, to limit the peak amplitude of the distended signals not larger than A_c and k should not be a negative real number. Therefore, k is confined to the interval (0, 1).

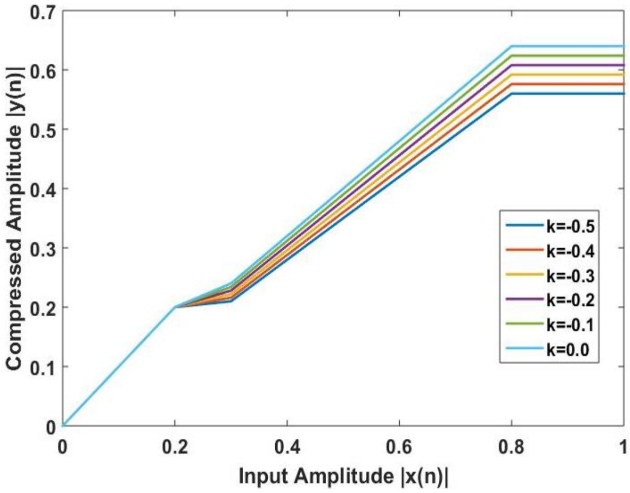

Figure 2: Compressed Amplitude for various values of k

In figure 2, original and compressed signal is shown for various values of k, while considering A_i=0.2 and Ac=0.8. Therefore, for lower values A_i < 0.2, the input and output signals are linearly related and they are same. Similarly, for higher values Ac > 0.8, the output is constant and is at fixed value of 0.8. In between 0.2 < x < 0.8, the linear compression is achieved.

4. SIMULATION RESULTS

OFDM simulation to obtain PAPR is done in MATLAB, and list of parameters used in the simulation is detailed in Table 1.

Table 1: List of simulation parameters

List of Parameters	Value
Number of Symbols (N)	64-2048
Guard interval	N/4
Modulation	QAM
Channel Length	2 and 4
Number of iterations	10000
Signal to noise ratio	1 to 15

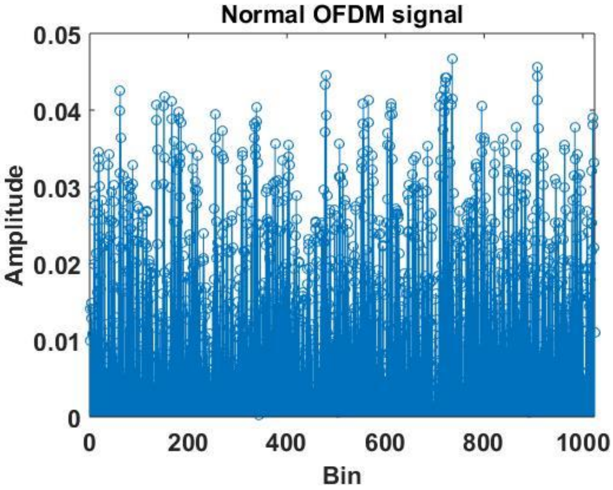

Figure 3: Normal OFDM signal plot in terms of amplitude vs. bin

In figure 3, normal OFDM signal plot in terms of amplitude vs. bin is shown while considering N=256. Here, a large variation in amplitude is observed. The maximum observed value is 0.0466 and minimum observed value is 2.92×10^{-4}. To tackle this large variation clipping is used and obtained results are shown in figure 4. Here, if particular bin value is higher than 0.8×maximum amplitude value than it is bring down to 0.8×maximum amplitude value, thus bin values above this are clipped. Now the maximum value is 0.0373.

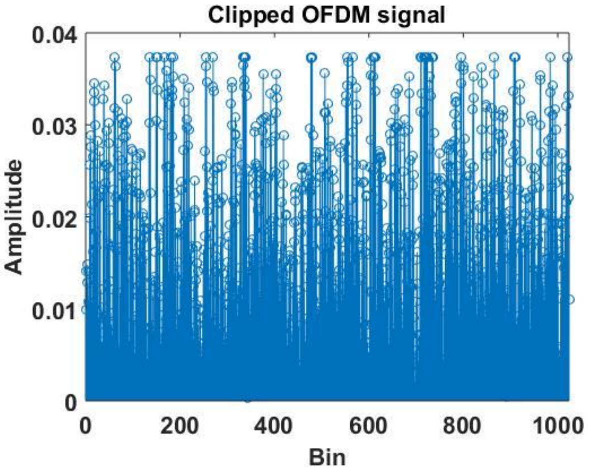

Figure 4: Clipped OFDM signal plot in terms of amplitude vs. bin

In figure 5, companded OFDM signal is shown, with maximum value of 0.0373, and minimum value of 0.0336. Therefore, variation in amplitude reduces significantly.

In figure 6, expanded OFDM signal is shown, which is exactly similar to clipped OFDM signal. However, there may be some differences which are observed by human eye. In the similar context difference between clipped and expanded OFDM signal is plotted in Figure 7. Here it can be observed that differences are negligible. Therefore, it ca inferred that error-free compression and expansion is possible. However, due to clipping of the signal distortion can be found between original OFDM signal and expanded OFDM signal, as plotted in figure 8. This type of distortion is known as clipping distortion.

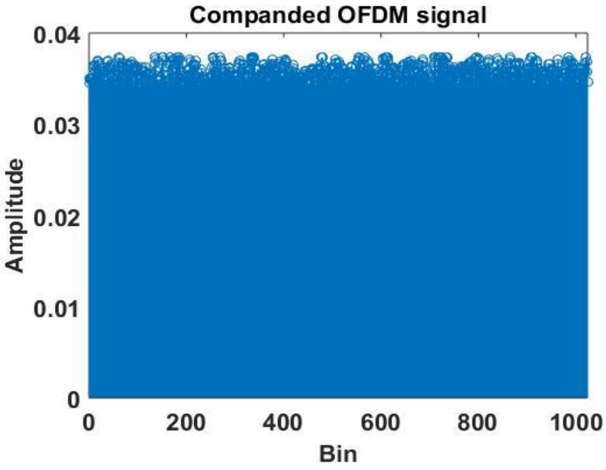

Figure 5: Companded OFDM signal plot in terms of amplitude vs. bin

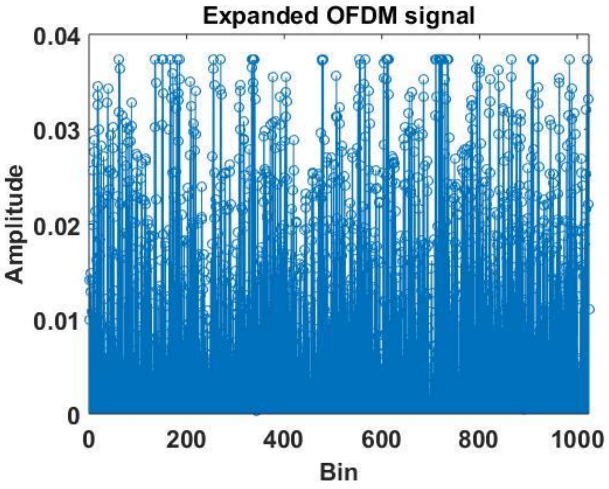

Figure 6: Expanded OFDM signal plot in terms of amplitude vs. bin

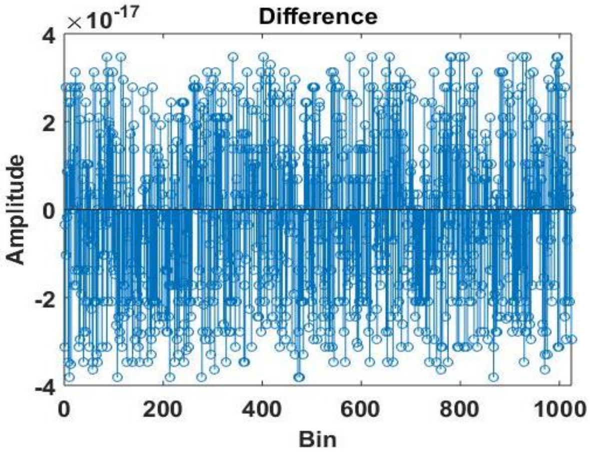

Figure 7 Difference between compressed and expanded signal in terms of amplitude vs. bin

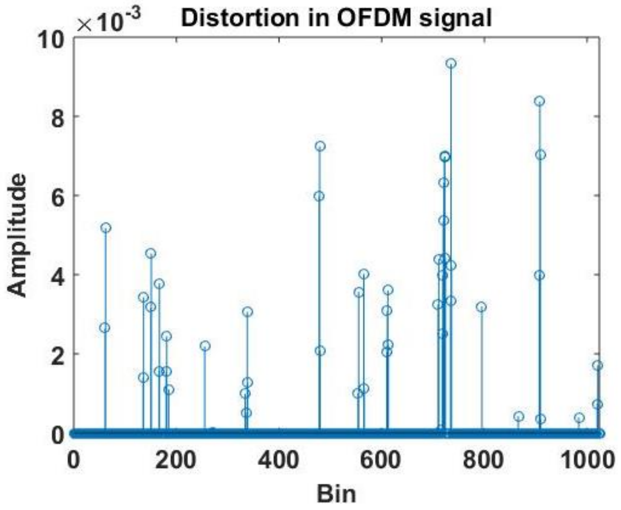

Figure 8 Difference between original and expanded signal in terms of amplitude vs. bin

The results are also obtained for other values of k and PAPR in dB are shown in Table 1. For each value of k, PAPR is obtained under three cases;

1. PAPR of original OFDM (dB)
2. PAPR of clipped OFDM (dB)
3. PAPR of Non-linear Companding OFDM (dB)

It is customary to note that Monte-carlo simulation is performed for N×L, values. Thus, this number is 1024, which is very less in comparison to steady state values. Therefore, in different set of simulations variations in curve and obtained PAPR can be observed. However, particular k value, results are obtained for similar set of 1024 values.

Table 1: Comparison of different schemes N=256, L=4

k	PAPR of original OFDM (dB)	PAPR of clipped OFDM (dB)	PAPR of Non-linear Companding OFDM (dB)
0.1	14.94	10.73	0.96
0.2	14.94	10.73	1.95
0.3	14.94	10.73	2.98
0.5	14.94	10.73	5.12

Table 2 Comparison of different schemes k=0.1, L=4

N	PAPR of original OFDM (dB)	PAPR of clipped OFDM (dB)	PAPR of Non-linear Companding OFDM (dB)
64	16.86	12.56	1.08
128	17.15	12.85	1.08
256	14.94	10.73	0.96
512	20.76	16.36	1.25
1024	20.20	15.80	1.22
2048	20.01	15.67	1.22

It is clear from the table that, the proposed scheme out-performs the previous schemes to a great extend.

5. BER ANALYSIS

We carry out an additional task which is termed as Companding transform after completing the adjustment of

OFDM signals. This produces companding distortion. Henceforth, how to lessen the effect of companding distortion on the BER execution is the important factor in planning companding transform. In this part of the article, we derived a typical plan benchmark for companding transform in order to minimize companding distortion on the basis of the theoretical research made on the BER performance as far as companding distortion is concerned. [22]

Discussing about companding transform, the companded signal y_n is considered as the basic OFDM signal x_n in addition to an added substance companding distortion signal c_n. At that point, y_n can be defined as

$$y_n = x_n + c_n \tag{6}$$

In the above equation, C_n and X_n have the same phase. On the basis of the above equation, we can define the y_n power as

$$\begin{aligned} \sigma_y^2 &= E\left(y_n^* y_n\right) = E\left(\left(x_n + c_n\right)^* \left(x_n + c_n\right)\right) \\ \sigma_y^2 &= \sigma_x^2 + 2E\left(c_n^* x_n\right) + \sigma_c^2 \end{aligned} \tag{7}$$

In this, $\sigma_c^2 = E\left(c_n c_n^*\right)$ is C_n power. Due to the fact that we have a constant average signal power in companding process, we have

$\sigma_x^2 = \sigma_x^2 + 2E\left(c_n^* x_n\right) + \sigma_c^2$, or we have,

$$\sigma_c^2 = -2E\left(c_n^* x_n\right) \tag{8}$$

In mathematics, the Bussgang theorem is a theorem of stochastic analysis. The theorem states that the crosscorrelation of a Gaussian signal before and after it has passed through a nonlinear operation are equal up to a constant.

On the other side, on the basis of the Bussgang Theorem, we can classify the companded signal into two parts i.e. attenuated signal and uncorrelated signal. Hence, we can rewrite y_n as

$$y_n = a x_n + d_n \tag{9}$$

Here, the parameter α is representing attenuating factor. We can make the calculation of α by

$$\alpha = \frac{E\left(y_n^* x_n\right)}{E\left(\left|x_n\right|^2\right)} = 1 + \frac{E\left(c_n^* x_n\right)}{\sigma_x^2} \tag{10}$$

By submitting (8) equation into (10), we have

$$\alpha = 1 - \frac{\sigma_c^2}{2\sigma_x^2} \tag{11}$$

After the y_n companded signal passed the AWGN channel, we can express the received signal by

$$r_n = y_n + \omega_n = \alpha x_n + d_n + \omega_n \tag{12}$$

In the above equation, the parameter ω_n represent the additive white Gaussian variable, while the variance of ω_n is denoted by σ_ω^2. Now, we can have the recovered signal by

$$\hat{x}_n = \frac{r_n - d_n}{\alpha} = \frac{y_n + \omega_n - d_n}{\alpha} = x_n + \frac{\omega_n}{\alpha} \tag{13}$$

At this point, the SNR at the receiver is

$$SNR = \frac{\left|\alpha\right|^2 \sigma_x^2}{\sigma_\omega^2} = \left(1 - \frac{\sigma_c^2}{2\sigma_x^2}\right)\frac{\sigma_x^2}{\sigma_\omega^2}. \tag{14}$$

The values of σ_x^2 is the variance of original OFDM signal, and σ_c^2 is the variance of difference of original and companded signal. These values can be obtained from above simulations. For the case of N=256, the value of σ_c^2 is 7.38×10^{-7}, and the value of σ_x^2 is 9.46×10^{-5}.

Therefore, the SNR is

$$SNR = \frac{\left|\alpha\right|^2 \sigma_x^2}{\sigma_\omega^2} = 0.9922\frac{\sigma_x^2}{\sigma_\omega^2}.$$

Or in other words:

$$SNR_{New}(dB) = -0.08 + SNR_{old}(dB)$$

Similarly, for other values of *k* and *N* calculations can be done. The BER performance for original OFDM signal under BPSK modulation scheme is shown in Figure 9. Here, well below 10 dB SNR, a bit error rate of 10^{-5} is achievable. For the proposed companding scheme same results are also plotted and it has been found that they are in well agreement, therefore with proposed scheme, PAPR is reduced significantly while we have maintained BER performance.

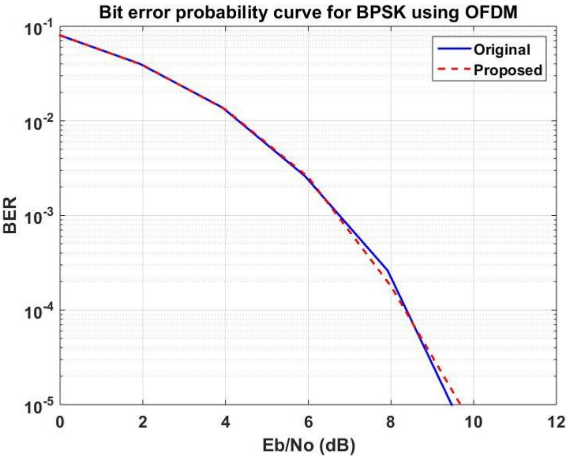

Figure 9 BER vs. Eb/No under original and proposed schemes (BPSK)

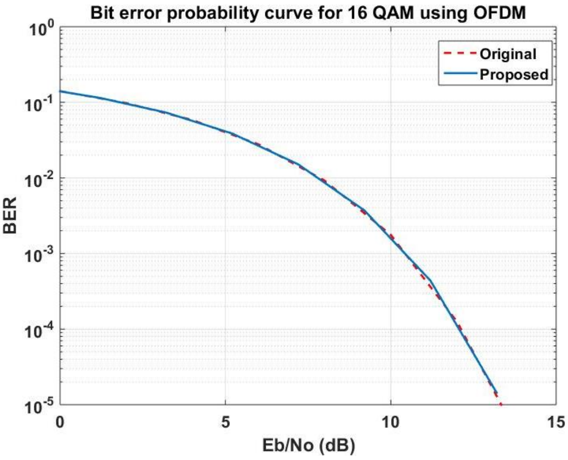

Figure 10 BER vs. Eb/No under original and proposed schemes (16 QAM)

The BER performance for original OFDM signal under 16 QAM modulation scheme is shown in Figure 10. Here, well below 15 dB SNR, a bit error rate of 10^{-5} is achievable. For the proposed companding scheme same results are also plotted and it has been found that they are in well agreement, therefore with proposed scheme, PAPR is reduced significantly while maintaining BER performance under various modulation schemes.

6. CONCLUSIONS

In this paper, a new piecewise linear companding scheme is proposed aiming at mitigating companding distortion to maintain the BER performance. Based on the theoretical analysis of the BER performance in terms of companding distortion, we get the general design criteria for companding transform that companding transform should avoid unnecessary compression and expand larger signals with smaller amplitude increments. Based on the design criteria, we propose a new piecewise linear companding scheme. By carefully designing the companding parameters, the proposed scheme can effectively reduce companding distortion.

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