

## Distance Preserving Mapping and Simple Intelligent Decoder for Modified Linear Block Code

<sup>1</sup>Seema Prashant Talmale, <sup>2</sup>Srija Unnikrishnan and <sup>3</sup>B. K. Lande

<sup>1</sup> Faculty K. J. Somaiya College of Engineering and Research Scholar, Department of Electronics Engineering, Fr. C. Rodrigues College of Engineering Bandra (W), Mumbai 400 050 India.

<sup>2</sup>Department of Electronics Engineering, Fr. C. Rodrigues College of Engineering Bandra (W), Mumbai 400 050 India.

<sup>3</sup>Department of Electronics Engineering, Datta Meghe College of Engineering, Navi Mumbai, India.

E-mail: <sup>1</sup>seematalmale@somaiya.edu, <sup>2</sup>srija@fragnel.edu.in, <sup>3</sup>bklande@gmail.com

### ABSTRACT

This article proposes an idea regarding the generation distance preserving mapping (DPM) from the solution set of discrete-time state equation. A modified linear block code is proposed from such DPM for which the distance preserving property is observed. The designed code is generalized from the  $n^{\text{th}}$  order solution space. As the code words of such established codes are very unique with regards to their bit positions, it is easy to solve the decoding problem at the receiver end. This correspondence also proposes an Intelligent Decoder (ID) along with the construction of decoding algorithm. Also, the methodology is developed for increasing the minimum distance between the codewords for the codes designed using DPM. The performance of the designed codes has been carried out using simulation.

**Keywords:** Generator matrix, Discrete-time system, Bounds, Minimum distance, Codeword, BER (Bit Error Rate)

### 1. INTRODUCTION

The research in the field of error-correcting codes these days has got a lot of importance. More particularly, the progress in this research field is possible due to progress in the field of discrete mathematics. It is well known that established subspace of given space can be represented in the form of codes. So, the coding theory is more related to space theory. According to Rosenthal [1], the connection between linear systems and error-correcting codes can be established. He related in a very elegant way the linear systems which are expressed with the help of equations directly to the construction of error-correcting codes. Rosenthal [2] explained that for the applications such as deep space communication for transmission of pictures and other information, NASA used the error-correcting codes very efficiently. Rosenthal [2] also gives the various problems associated in the field of Codes such as lack of decoding algorithms for the codes with higher Mcmillan degree. He also gives the outlook to the system which is described by  $x(k+1) = Ax(k) + bu(k)$ . According to him state-space realisation of such systems can be used to define the codes. Rosenthal [3] puts up some interesting problems in the system theory. He explained about code construction and the difficulty in achieving the free minimum distance between the code words. The large value of the free minimum distance between the codewords is very much desirable for error-correcting codes to incorporate the good error correction capability.

Kalman [4] gives the idea of the solution space of discrete-time linear systems. According to [4], the initial state of linear systems described by  $[1111\dots1_{n-1}]$  can be returned to zero  $[0000\dots0_{n-1}]$  in the  $n^{\text{th}}$  sampling interval. Akbari and Gillespie [5] illustrate some techniques for increasing the free distance between the code words. [5] discusses more on frequency permutation arrays and its use in power line communications. [5] also gives the construction of frequency permutation arrays which are helpful in achieving the better distance increasing capabilities of the error-correcting codes.

Shao Xia and Zhang Weidang [6], explains the simple method for the realization of shortening of turbo codes. It also gives an idea regarding variable length shortened turbo codes. Marc P. C. Fossorier and Shu Lin [7] gives soft-decision decoding method for linear block codes based on the available information of ordered statistics of the information set utilised during encoding procedure. Martin et al. [8] give soft input soft output decoding algorithm based on the calculation of extrinsic information generated from the list of codewords. Godoy et al. [9] discuss adaptive decoding of binary linear block codes. This is also information set based decoding. Walter Godoy and Emilio C. G. Wille [10] gives real-time modified information set based soft-decision decoding algorithm for block codes. Brante et al. [11] give decoding algorithm which has performance very close to maximum likelihood decoding. Guo et al. [12] developed an information set dependant decoding algorithm and applied

it to the cryptographic application. Chang et al. [13] discuss an efficient decoding algorithm for constant composition codes developed from distance increasing (or distance preserving) mapping. Theo G. Swart and Hendrik C. Ferreira [14] developed decoding methodology for decoding of DPM used in power line communication.

In this article, a new idea regarding the generation of modified linear block code from the solution set of discrete-time state equation is proposed. Such codes are established from the part of the solution space of the given discrete-time system. This is more advantageous as the idea is to use the part of the solution space and not the complete set of solution space. This fact of an idea helps in increasing the minimum distance between the code words. With this methodology, for higher-order codes, distance preserving characteristics is observed. So, an intelligent decoder/pattern generator is proposed which has complete information regarding the various patterns associated with the transmitted codewords. An idea of permutation is used further to obtain the new basis vectors of the generator matrix of the established code. Such DPM based permutation codes have application in power line communication. Decoding of such codes is easier as positions of information bits in the generated codewords are fixed. In the later section of this paper, we propose a decoding algorithm for such modified linear block codes developed using DPM.

The paper is organised as follows: Section 2 discusses the designing of the code for the linear system. Section 3 gives the construction of DPM. In section 4, of this correspondence, we design an intelligent decoder for the modified linear block codes developed using earlier constructed DPM. In section 5, we propose a decoding algorithm for the established code. Section 6 gives the system model for the entire system of this research work. In section 7, we discuss the mathematical results of the analytical research work done in this study. In section 8, we propose the methodology to increase the minimum distance between the codewords of the established codes. Section 9 gives the application area of this work. Paper is concluded in section 9.

## 2. DESIGNING THE CODE FOR THE LINEAR SYSTEM

Consider the Discrete-time system as described by the following expression:

$$x(k+1) = Ax(k) + bu(k).$$

Assuming the initial conditions for the above system as [1 1 1 1 1] which is a fifth-order system. When we try to find the solutions for this system at the various instant of time, it is well known that the system will be steered to origin at the  $n^{\text{th}}$  instant of sampling as discussed in [4] and [15-17].

Thus, the various solutions are [1 1 1 1 0; 1 1 1 0 0; 1 1 0 0 0; 1 0 0 0 0; 0 0 0 0 0]. For the fifth-order system, we get four solutions. If we consider the above solution space as basis vectors and obtain the generator matrix  $G = [I: P]$ , where P is the basis vector and I is an identity matrix.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The devised P matrix is of an upper triangular matrix in nature. The modified linear block codes established from such a generator matrix is discussed in [18 and 19]. It is observed that the established code from the above system with the help of basis vectors is an (n, k) that is (10, 5) where n=10, k=5, modified linear block code with the minimum distance between the codewords as 2.

In this correspondence, we propose to form the generator matrix with the help of the solution space by skipping the last solution. It is observed that when codewords are obtained from such an established generator matrix, the minimum distance between the codewords increases [20]. So the generator matrix will become:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Now, the established code is (9, 4) code. The minimum distance between the codewords for (9, 4) code is found to be three. This method of establishing the block code can be generalised to any (n, k). Such an established code has distance preserving characteristics. For example, next (n, k) code in this series will be (11, 5) code. The generator matrix for (11, 5) modified block code is as follows:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The minimum distance between the codewords for (11, 5) modified linear block code is again found to be three. In the next section of this correspondence, we propose distance preserving mapping (DPM) for the established block code.

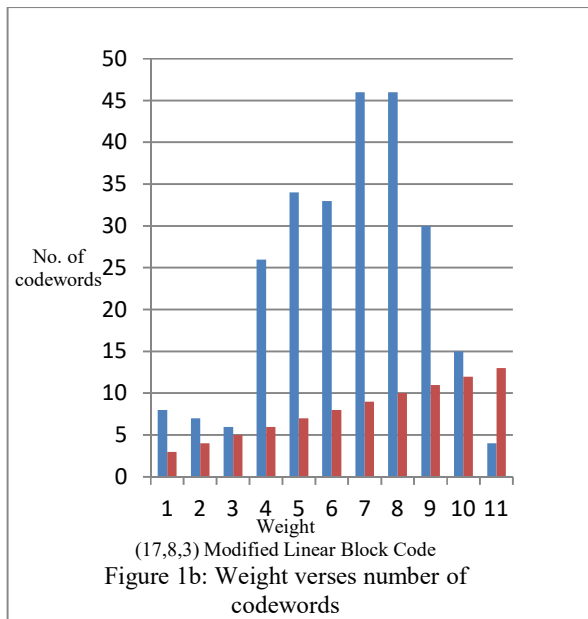
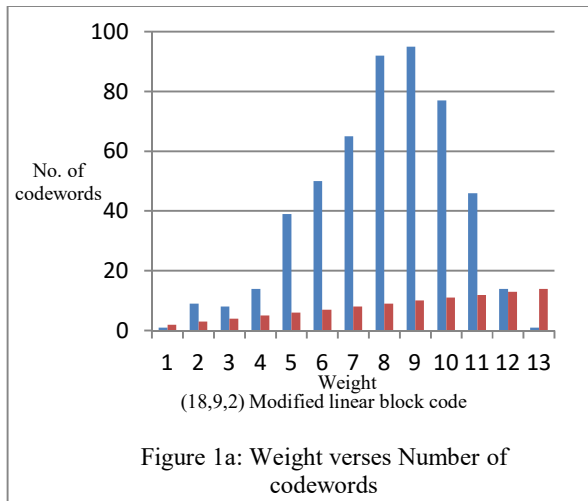
## 3. DISTANCE PRESERVING MAPPING (DPM)

Let the order of upper triangular matrix (UTM) = x  
Construction: Mapping Algorithm  $\psi$

©2012-19 International Journal of Information Technology and Electrical Engineering

Input :  $UTM(C_1, C_2, \dots, C_n) \in Z_2^n$   
 Output :  $(\Omega_1, \Omega_2, \dots, \Omega_n) \leftarrow \psi UTM(C_1, C_2, \dots, C_n)$   
 Begin :  $\psi UTM(C_1) \leftarrow (\Omega_1, \Omega_2, \dots, \Omega_n) \leftarrow$  First row of  $UTM$  as it is.  
 For  $i = 0$  to  $n$  do  
 Consider only  $(x-1)$  rows for  $UTM(C)$   
 Merge  $(x-1)^{th}$  order Identity Matrix to  $UTM(C)$   
 Get the Generator matrix  $G$   
 end.

The above proposed DPM is applied to some higher-order codes. For example (17, 8, 3) modified linear block code is obtained from (18, 9, 2) code as suggested in [20]. Following weight distribution characteristics is observed for (17, 8, 3) and (18, 9, 2) code.



#### 4. DESIGNING OF PATTERN GENERATOR / INTELLIGENT DECODER (ID)

The code words obtained from designed  $(n, k)$  code are very unique with regards to their position of one's and zero's. So, it is very easy to train the decoder with the intelligent information about the pattern of various code words with regards to the position of one's at different location of the codeword, count of minimum and the maximum number of one's in the received  $(n, k)$  codeword. This training to the decoder could be explained as follows: Step 1: It is obvious that the decoder must have information regarding how much is the length of the codeword which is being transmitted that is it must have information regarding  $(n, k)$  parameters of the code. It will also know that there will be a total of  $2^k$  code words. Out of the  $2^k$  codewords, one code word will be all zero code word. If the codeword is represented as follows:

$$\left\{ \underbrace{MSB, MSB + 1, \dots, k}_{k \text{ part of code word}}, \underbrace{k + 1, k + 2, \dots, n}_{\text{remaining part of the code word}} \right\} \dots \dots \dots (1)$$

that is in other words

$$\left\{ \underbrace{1^{st} \text{ bit}, 2^{nd} \text{ bit}, \dots, k^{th} \text{ bit}}_{k \text{ part of code word}}, \underbrace{(k + 1)^{th} \text{ bit}, (k + 2)^{th} \text{ bit}, \dots, n^{th} \text{ bit}}_{\text{remaining part of the code word}} \right\}$$

then there will be  $k$  number of codewords with single one in the  $k$  part of the codeword, as these code words are directly obtained from the generator matrix of the designed code.

Step 2: Depending upon the information available about  $(n, k)$ , the intelligent decoder will first keep ready the count of the maximum number of ones which is always obtained in code word which is established by adding all  $(k-1)$  rows of the generator matrix of the code. In this way, this code will have maximum number of one's =  $3 + (k-1) + (n-k-3)/2 = (n+k+1)/2$ .

Step 3: Similarly, ID will also be aware of the minimum number of ones for this  $(n, k)$  code = 3 as it is a single bit error-correcting code with the minimum distance as 3 which is equal to the weight of the codeword. Also from the geometry of the generator matrix of the  $(n, k)$  code, it is clear that the minimum number of one's for this code = 3.

Step 4: There will be in all  $k$  number of codewords with the number of one's in it equal to 3. Out of these  $k$  number of codewords  $(k-1)$  code words with total number of ones as 3 in it are obtained by adding two codewords at a time directly from the established generator matrix of the the designed  $(n, k)$  code and the remaining code word with total number of one's as 3 is obtained always as the last codeword of the established generator matrix of the  $(n, k)$  code.

Step 5: About the pattern of the code words which are directly obtained from the generator matrix of the designed code:

There will be always the presence of single one in k part of the code words which are obtained as mentioned above. Two patterns are observed for the code words obtained directly from the generator matrix and they are as follows: Pattern (A): If there is a presence of single one up to the k<sup>th</sup> location of the part of the codeword then there will be the presence of double one that is at (k+1)<sup>th</sup> and (k +2)<sup>th</sup> location of the codeword and rest of the bits of the part of the code word will be zero.

Pattern (B) : In general, if there is presence of x zero's at (k)<sup>th</sup>, (k - 1)<sup>th</sup>, (k - 2)<sup>th</sup>,.....location then there will be (2 + x) one's at (k +1)<sup>th</sup>, (k +2)<sup>th</sup>,.....location of the codeword and the remaining bits of the codeword are always zero. Example of the code words that could be obtained from (9, 4) code for Pattern (A) and Pattern (B) are illustrated with the help of following generator matrix for the designed (9, 4) code.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Step 6 : Apart from the codewords directly obtained from generator matrix, there are code word patterns which can be obtained by adding either even number of codewords of generator matrix-like by adding two, four, .....even number of codewords from the generator matrix or there is presence code word patterns which can be obtained by adding odd number of codewords of Generator matrix-like by adding three, five, .....(odd) number of codewords from the generator matrix at a time. The intelligent decoder could be trained for knowing the pattern of code words when such codewords are established by adding an even number of codewords from the generator matrix is explained in step 7.

Step 7: For the designed (n, k) code, there are total  $C_{\epsilon}^k$  number codewords obtained by adding even number of codewords from the generator matrix, where  $\epsilon$  = Even number of addition of codewords at a time from generator matrix.

There are three patterns observed in  $C_{\epsilon}^k$  combination and they are  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  as described below:

(1)  $\lambda_1$  pattern in  $C_{\epsilon}^k$  : If there is presence of "ε" even number of one's side by side in "k" part of the codeword as shown in equation (1) which are actually obtained by adding "ε" even number of code words from Generator matrix then the code word will have 101010 pattern up to "ε" times starting from n<sup>th</sup>, (n - 1)<sup>th</sup>,.....location of the codeword and the rest of the bits of such codeword are always zero.

(2)  $\lambda_2$  pattern in  $C_{\epsilon}^k$  : If there is presence of two one's which are separated by "x" number of zero's in "k" part of the codeword as mentioned in equation (1) then there will be (x + 1) one's starting from n<sup>th</sup>, (n - 1)<sup>th</sup>,.....location of the

codeword and the rest of the bits of such codeword are always zero.

(3)  $\lambda_3$  pattern in  $C_{\epsilon}^k$  : Apart from the main  $\lambda_1$  and  $\lambda_2$  patterns  $\lambda_3$  pattern is also observed in the code words obtained from generator matrix which is nothing but the rotation of  $\lambda_1$  and  $\lambda_2$  pattern of k part of the codeword from left to right. For this, there will also be a rotation of  $\lambda_1$  and  $\lambda_2$  pattern from right to left that is from n<sup>th</sup> part of the codeword towards (k + 1)<sup>th</sup> part of the codeword corresponding to the number of bits of the rotation in "k" part of the codeword.

Some important conditions observed in the codewords generated as follows:

(a) If zero's are there at MSB part of the "k" part of the codeword then there will be the same number of zero's at n<sup>th</sup>, (n - 1)<sup>th</sup>,.....location of the codeword.

(b) If zero separated pattern that is  $\lambda_2$  pattern is followed by  $\lambda_1$  pattern that is side by side one pattern is present in the "k" part of the codeword then the bits generated in n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword need to be separated by equivalent number of zero's between  $\lambda_2$  and  $\lambda_1$  pattern generated from n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword.

(c) If  $\lambda_1$  pattern is followed by  $\lambda_2$  pattern in the "k" part of the codeword then the bits generated in n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword need not to be separated by equivalent number of zero's between  $\lambda_2$  and  $\lambda_1$  pattern generated from n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword.

(d) It is also observed in some code word patterns that there could be the presence of a very similar attached pattern in the bits in "k" part of the codeword then  $\lambda_1$  and  $\lambda_2$  patterns generated from n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword should be separated by extra zero's. For example in (13,6) one of the codewords appears as follows:

$$\begin{array}{cc} \underline{101011} & \underline{0010011} \\ \text{k part of the code word} & \text{(k+1)th to nth part of the code word} \end{array}$$

(e) Zero's between two similar patterns if occurred in "k" part of the codeword then the same number of zero's should appear in between patterns of bits generated from n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword.

(f) Above condition "e" is also true for the appearance of zero's between even dissimilar patterns if occurred in "k" part of the generated codeword. For example in (17,8) code one of the codewords appears as follows:

$$\begin{array}{cc} \underline{00011011} & \underline{001001000} \\ \text{k part of the code word} & \text{(k+1)th to nth part of the code word} \end{array}$$

(g) This is the last condition observed in this category. If the similar pattern appears side by side in "k" part of the codeword then the bits generated from n<sup>th</sup> to (k + 1)<sup>th</sup> part of the codeword should be separated by a single zero. For example in (17, 8) code one of the codewords appears as follows:

$$\begin{array}{cc} \underline{1011} \underline{1001} \underline{0} & \underline{000111011} \\ \text{k part of the code word} & \text{(k+1)th to nth part of the code word} \end{array}$$

Step 8: For the designed (n, k) code, there are total  $C_{\phi}^k$  number codewords obtained by adding the odd number of

codewords from the generator matrix, where  $\varphi$  = the Odd number of addition of codewords at a time from generator matrix.

There are three patterns observed in  $C_{\varphi}^k$  combination and they are  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  as described below:

(1)  $\lambda_1$  pattern in  $C_{\varphi}^k$ : If there is presence of “ $\varphi$ ” odd number of one's side by side in “k” part of the codeword as shown in equation (1) which are actually obtained by adding “ $\varphi$ ” odd number of code words from Generator matrix then the code word will have 101010 pattern up to “ $\varphi - 1$ ” times starting from  $n^{\text{th}}$ ,  $(n - 1)^{\text{th}}$ ,....location of the codeword and the rest of the bits of such codeword are always one's.

(2)  $\lambda_2$  pattern in  $C_{\varphi}^k$ : If there is presence of two one's which are separated by “x” number of zero's in “k” part of the codeword as mentioned in equation (1) then there will be  $(x + 1)$  one's starting from  $n^{\text{th}}$ ,  $(n - 1)^{\text{th}}$ ,....location of the codeword and the rest of the bits of such codeword are always one.

(3)  $\lambda_3$  pattern in  $C_{\varphi}^k$ : Apart from the main  $\lambda_1$  and  $\lambda_2$  patterns  $\lambda_3$  pattern is also observed in the code words obtained from generator matrix which is nothing but a rotation of  $\lambda_1$  and  $\lambda_2$  pattern of k part of the codeword from left to right. For this, there will also be a rotation of  $\lambda_1$  and  $\lambda_2$  pattern from right to left that is from  $n^{\text{th}}$  part of the codeword towards  $(k + 1)^{\text{th}}$  part of the codeword corresponding to the number of bits of the rotation in “k” part of the codeword.

Some important conditions observed in the code words generated as follows:

- (h) If zero's are there at MSB part of the “k” part of the codeword then there will be the same number of zero's at  $n^{\text{th}}$ ,  $(n - 1)^{\text{th}}$ ,....location of the codeword.
- (i) If there is a presence of single one at “k<sup>th</sup>” location of the code word then there is a presence of double one's at  $(k + 1)^{\text{th}}$  and  $(k+2)^{\text{th}}$  location.
- (j) If there is a presence of single zero at “k<sup>th</sup>” location of the code word then there is a presence of triple one's at  $(k + 1)^{\text{th}}$  and  $(k+2)^{\text{th}}$  and  $(k+3)^{\text{th}}$  location.
- (k)  $\lambda_1$  pattern of these code words goes independently.
- (l)  $\lambda_2$  pattern of these code words goes along with the conditions given by (a), (b), (c), (d), (e), (f), (g), (h), (i) and (j).

## 5. DECODING ALGORITHM

Step 1: On receiving the codeword decoder will first count the number of ones in it. If the number of ones in the received codeword are 2 then the decoder will compare it with all the code words of the pattern having the number of ones in it equal to 3 available with the intelligent decoder as it is a single bit  $(n, k)$  error-correcting code. A nearest matching pattern will be the actually transmitted codeword.

Step 2: Similarly, if the number of ones in the received codeword are  $\leq x$  then the decoder will compare it with all the code words of the pattern having the number of ones in it equal to  $(x-1)$  available with the Intelligent Decoder. A nearest matching pattern will be the actually transmitted codeword.

Step3: Else the decoder will compare it with all the code words of the pattern having the number of ones in it equal to  $x$  available with the intelligent decoder. A nearest matching pattern will be the actually transmitted codeword.

Step 4: Else the decoder will compare it with all the code words of the pattern having the number of ones in it equal to  $(x+1)$  available with the intelligent decoder. A nearest matching pattern will be the actually transmitted codeword. Stop here.

## 6. SYSTEM MODEL FOR GENERATION AND DETECTION OF MODIFIED LINEAR BLOCK CODE DEVISED THROUGH DPM.

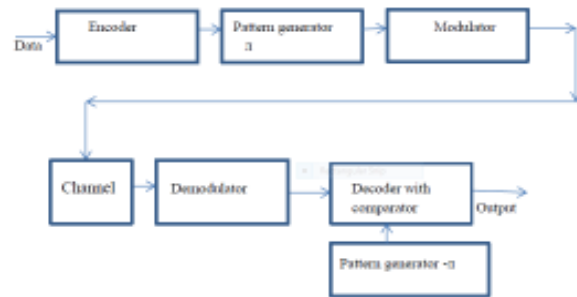


Figure 2: System Model

## 7. MAIN RESULT

Lemma 1: The part of the solution space of the given discrete-time system  $x(k+1) = Ax(k) + Bu(k)$  whose initial condition is  $[1 \ 1 \ 1 \dots \ 1_{n-1}]$  when used to form the generator matrix  $G = [I : P]$ , establishes the modified Block code with minimum distance greater than or equal to three.

*Remark 1:* The procedure discussed for the generation of the code using the part of the solution space itself proves that the minimum distance between the codewords is three or greater than three.

Lemma 2: For a  $n^{\text{th}}$  order given upper triangular matrix UTM(C), UTM(C) to  $\Psi$ UTM(C) mapping which is established by considering  $(n-1)$  rows only of UTM(C), if generator matrix tried to obtain using  $G = [I : \Psi$ UTM(C)] then a code established will observe a distance preserving mapping.

*Remark 2:* When the generator matrix is established by UTM(C) to  $\Psi$ UTM(C) mapping such that  $G = [I :$

$\Psi\text{UTM}(C)$ ], where  $I$  is an identity matrix, the codeword from the last row of generator matrix will always have minimum weight equal to three. From [21], for a linear code, the minimum distance  $d$ , which satisfies,  $d^* = \min \{w(c)\} = w^*$ , where the minimum is overall codewords except the all-zero codeword,  $w(c)$  = weight of the codeword. So, in general, such mapping will observe DPM.

### 8. METHODOLOGY TO INCREASE THE MINIMUM DISTANCE OF THE CODE

Here we discuss some more result on increasing the distance of the code more specifically about [17, 8] code whose minimum distance between the codewords is 5.

When  $x(k+1) = Ax(k) + bu(k)$  is solved whose initial condition is [1 1 1 1 1 1 1 1], it is observed that at 9<sup>th</sup> sampling state, the considered system turns to the origin [0 0 0 0 0 0 0 0]. Thus, there are eight basis vectors which will form the solution for the above-mentioned system. If the first seven solutions are used along with the initial condition to form the basis vectors of the generator matrix, the code which is established is [17,8] code. When experimented using MATLAB to find the minimum distance between the codewords of such established [17, 8] code, is found to be three. In order to increase the minimum distance between the code words, the first row of the established P part of the generator matrix is transformed as below:

$R_1 = R_1 + R_2 + R_3 + R_5 + R_6 + R_7 + R_8$ . Thus, the first row of the transformed P part of  $G_{NEW}$  is [1 1 0 1 0 1 1 0 1]. When the permutation is performed of this first row of P part of  $G_{NEW}$ , the established  $G_{NEW}$  is:

$$G_{NEW} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

When the code words are established from  $G_{NEW}$ , the minimum distance between the codewords of this new [17, 8] code is found to be 5. Performance of this [17, 8,  $d_{min}=5$ ] is evaluated in MATLAB. Performance is found to be better than [17, 8,  $d_{min}=3$ ] code as the minimum distance between the codewords is increased through the permutation process. This comparison is illustrated in figure 3.

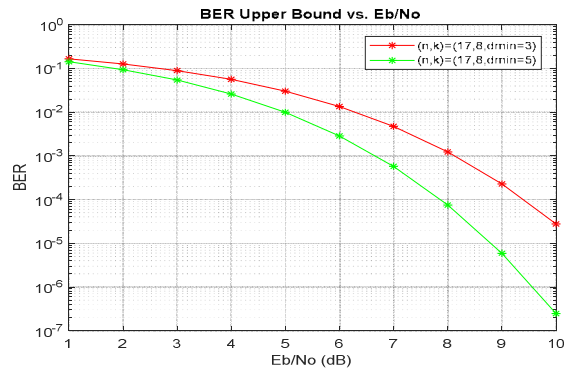


Figure 3: Comparison of performance of [17, 8,  $d_{min}=3$ ] and [17, 8,  $d_{min}=5$ ] code.

### 9. APPLICATION

The report mentioned by CCSDS (The Consultative Committee for Space Data System) [22], gives five application profiles places which demand on the coding system. According to the trade study of CCSDS for Mode A (Deep space emergency communication, unmanned missions) and Mode B (command and ARQ which is similar to the traditional Tele-command problem) must meet  $CWER < 10^{-3}$ ,  $P_u < 10^{-9}$ , and  $E_b/N_0$  as small as possible. Mode A and Mode B applications of this report suggest the use of lengthy codes with code rate less than 0.5 and must possess high minimum distance characteristics, which is possible with the idea suggested in this paper.

### CONCLUSION

An idea regarding the generation of modified linear block code from the solution set of discrete-time state equation is proposed here. The procedure described here can be further extended to higher-order discrete-time systems and lengthier codes for which distance preserving property is observed. The decoding algorithm is as well developed for devised modified linear block code by taking the advantage of the fixed position of the bits in the codewords. Such codes can be used in power line communication where constant weight codes are desired. The crucial idea of permutation is explored further to establish the basis vectors of the generator matrix which is useful in gaining the better distance characteristics of the established codes. According to CCSDS studies, such lengthier codes with good distance properties are in demand by space research systems.

### REFERENCES

[1] Joachim Rosenthal, "Connection between Linear Systems and Convolutional Codes", Springer-Verlag New York Inc. 2001.

- [2] Joachim Rosenthal, "An optimal control theory for systems defined over finite rings", Open Problems in Mathematical Systems and Control Theory Chapter 38, pages 192-201, Springer Verlag, 1998.
- [3] Joachim Rosenthal, "Some Interesting Problems in Systems Theory which are of Fundamental Importance in Coding Theory", Proceedings of the 36<sup>th</sup> Conference on Decision & Control San Diego, California USA December 1997.
- [4] R. E. Kalman, "On the general theory of Controls System", in the Proceedings of the First IFAC Moscow Congress by Butterworth Scientific Publications in 1960.
- [5] Marzieh Akbari, Neil I. Gillespie, "Increasing the minimum distance of codes by twisting", the electronic journal of Combinatorics 25(3) (2018), P3.36.
- [6] Shao Xia and Zhang Weidang, "Shortening the Turbo Codes Based on Unequal Error Protection", International Journal of Multimedia and Ubiquitous Engineering Vol.10, No.8 (2015), pp.73-82.
- [7] Marc P. C. Fossorier and Shu Lin, "Soft-Decision Decoding of Linear Block Codes Based on Ordered Statistics", IEEE Transactions on Information Theory, vol. 41, no. 5, September 1995.
- [8] Philippa A. Martin, Desmond P. Taylor, Marc P. C. Fossorier, "Soft-Input Soft-Output List-Based Decoding Algorithm", ISIT 2002, Lausanne, Switzerland, June 30 -July 5, 2002.
- [9] Walter Godoy, Emilio Carlos Gomes Wille and Joao Augusto Teixeira da Cunha, "Adaptive Decoding of Binary Linear Block Codes Using Information Sets and Erasures", 2010 Third International Conference on Communication Theory, Reliability, and Quality of Service.
- [10] Walter Godoy and Emilio C. G. Wille, "A Simple Acceptance Criterion for Binary Block Codes Soft-Decision Algorithms", IEEE International Conference on Internet and Web Applications and Services, 2006.
- [11] G. G. de O. Brante, D. N. Muniz and W. Godoy, "Information Set Based Soft-Decoding Algorithm for Block Codes", IEEE Latin America Transactions, vol. 9, no. 4, July 2011.
- [12] Qian Guo, Thomas Johansson, Erik Mårtensson and Paul Stankovski, "Information Set Decoding with Soft Information and some cryptographic applications", IEEE International Symposium on Information Theory, 2017.
- [13] Jen-chun Chang, I-te Tsai, Hsin-lung Wu, "Efficient decoding algorithm for constant composition codes", ISITA2010, Taichung, Taiwan, October 17-20, 2010.
- [14] Theo G. Swart and Hendrik C. Ferreira, "Decoding Distance-preserving Permutation Codes for Power-line Communications", IEEE, 2007.
- [15] Agashe S.D., Lande B. K., "A New Approach to State-transfer Problem", J. Franklin Inst., Vol. 333(B), No. 1, pp. 15-21, (1996).
- [16] B.K.Lande, "Some General Problems in Control Theory", A PhD thesis submitted to Indian Institute of Technology, I.I.T., Bombay, 1984.
- [17] Vinit Kotak, "Design of a Code: A Systematic Approach", A PhD Thesis submitted to Veermata Jijabai Technological Institute, Matunga, Mumbai, 2015.
- [18] Seema Talmale, Srija Unnikrishnan, B.K.Lande, "A Modified Block Code using Controllability of Linear System", ICAC3, 2017.
- [19] Seema Talmale, Srija Unnikrishnan, B.K.Lande, "Modified Linear Block Code with Code Rate 1/2 and Less than 1/2", Journal of Discrete Mathematical Sciences and Cryptography, pp. 139 – 150, March 2019.
- [20] B. Honary, G. Markarian, "New simple encoder and trellis decoder for Golay codes", Electronics Letters 9th December 1993 Vol. 29 No. 25.
- [21] S.Lin and D. Costello, "Error Control Coding: Fundamentals and Applications", Prentice-Hall, Englewood Cliffs, NJ, 1983.
- [22] Report Concerning Space Data System Standards, Informational Report CCSDS 230.2-G-1 Green Book July 2014.

## AUTHOR PROFILES

**Seema Prashant Talmale** received her B. E. degree in 2001 and M.Tech. in 2010 from V.J.T.I., Matunga, Mumbai. She is working as an Asst. Professor in K.J.Somaiya College of Engineering, Vidyavihar Mumbai. She is currently pursuing her Ph.D. in Electronics Engineering. Her research interests are in Controls, Coding Theory and Communication Engineering.

**Dr. Srija Unnikrishnan** is the Principal of Fr. C. Rodrigues College of Engg., Bandra, Mumbai. She is currently member of the Adhoc Board of Studies of Electronics Engineering, University of Mumbai. She has completed her UG from Govt. College of Engg., Trivandrum, PG from Osmania University, Hyderabad and Ph.D. from University of Mumbai. She has academic experience of over 35 years and about 60 international and national level publications to her credit. She has supervised 30 M.E. projects and is presently guiding five research scholars. She has served in the programme committees of several International/National Conferences.

**Dr. B. K. Lande** received his B. E. degree in 1973 and M.E. in 1976. He received his Ph.D. degree in Control Theory from IIT Bombay. He worked as a Professor of Electrical Engineering at V.J.T.I., Matunga, Mumbai for 30 years. Presently, working as Professor of Electronics Engineering at Datta Meghe College of Engineering, Airoli, Mumbai. His interests are in Controls and Communication Engineering.