Transmission Penalty Analysis Due to Higher Order Dispersion and Optical Kerr’s Effect in Optical Heterodyne CPFSK Systems

1Anu Sheetal and 2Harjit Singh

1,2 Department of Electronics and Communication Engineering, Guru Nanak Dev University, Regional Campus, Gurdaspur, Punjab, India
E-mail: 1anusheetal.ecegsp@gndu.ac.in, 2hskahlon.ecegsp@gndu.ac.in

ABSTRACT

In this paper, theoretical analysis of a 40Gb/s optical heterodyne continuous-phase frequency shift keying (CPFSK) transmission system has been analyzed considering the effect of higher order dispersion (HOD) and self-phase modulation (SPM) in a nonlinear fiber medium. A general conversion matrix has been obtained describing phase and intensity transfer function between fiber input and output for laser source under the influence of HOD terms. The power transfer function and bit error rate (BER) are plotted for second-, third-, and fourth-order dispersion components and their combination for the phase-shift keying (DPSK), CPFSK, etc., the performance limitations of the fiber, it is imperative to calculate and compensate the second and higher-order dispersion effects as they are becoming increasingly significant in an optical link [1-4].

Optical continuous-phase frequency shift keying (CPFSK) with low modulation index (h≤1) and compressed spectrum is an attractive modulation scheme in high bit rate optical fiber transmission due to direct modulation potential of DFB laser [5]. However, for optical intensity modulated direct detection (IM–DD), the conversion of laser phase noise to intensity noise because of fiber dispersion may result in severe transmission penalty.

Compared to direct detection, the coherent receiver offers significant improvement in the receiver sensitivity and wavelength selectivity by adding the locally generated optical wave to the received signal and detecting the resultant sum. For various angle modulated transmission systems such as phase-shift keying (PSK), differential phase-shift keying (DPSK), CPFSK etc., the performance due to the cumulative effect of GVD, SPM and ASE noise is mostly assessed by numerical Split Step Fourier method. This approach provides precise simulation of the received waveform but requires vast amount of computation time [6-7]. The performance of these angle modulated systems depends significantly on various parameters like launch power, bit rate and spectral characteristics of the laser source. The laser phase noise is generally characterized in terms of the linewidth of the laser emission spectrum and is vital for the realization of a stable and reliable linewidth source.

Wang et al. computed the linewidth required by IM-DD systems near zero dispersion wavelengths from the relative intensity noise (RIN) for linear and dispersive optical fiber using a small-signal theory [8]. They analyzed the performance of optical communication systems with first-order dispersion using conversion matrix to obtain the fiber output in terms intensity and frequency modulation at fiber input. Crognale et al. further expanded the work of Wang by including the influence of second-order dispersion term describing the propagation of signal and noise through a lossless linear dispersive single mode fiber near zero dispersion wavelength [9]. For an ultrafast laser diode, they studied impact of second-order dispersion term on the modulation and noise properties using frequency response and the relative intensity noise (RIN). Further, Kaler compared large and small signal theory for intensity modulation response of semi-conductor lasers under the influence of the higher-order dispersion on dispersive optical communication system [10].

Min-Jong Hao et al. presented modified moments method for evaluating the bit error rates (BER) of coherent FSK and CPFSK systems considering the influence of intermediate frequency bandwidth and post detection filtering on the sensitivity performance of the system [11]. Further, Cartaxo et.al investigated the conversion of phase noise to intensity noise of laser and optical amplifier under the effect of fiber nonlinearities [12]. B. Pal analytically evaluated the transmission penalty due to group velocity dispersion, self-phase modulation, and amplifier noise in optical heterodyne CPFSK systems [13]. Also, S. P. Majumdar et al. evaluated the optical heterodyne CPFSK system impaired by self-phase modulation and polarization mode dispersion in a single mode fiber [14-18].
The frequency response for heterodyne CPFSK system including HOD terms has not been studied in previous work. Also, the relative intensity noise and BER analysis for CPFSK system with respect to linewidth of laser has not been evaluated so far. So, in this paper, we analytically evaluate the impact of linewidth of the laser source over received power and BER for varying values of modulation frequencies under the individual and cumulative effect of second-, third- and fourth-order dispersion terms in heterodyne CPFSK optical system. Also, the bit error rate performance and relative intensity noise of the system is evaluated for different values of linewidths and input power of the source. In Section 2, theoretical analysis of the CPFSK system has been carried out and a general conversion matrix is obtained giving phase and intensity transfer function between fiber input and output for laser source under the influence of HOD terms. In Section 3, results have been discussed for the CPFSK system and finally the conclusions are made in Section 4.

2. THEORETICAL ANALYSIS

In ultrafast long reach broadband optical communication systems, group velocity dispersion is a considerable restraining factor to deteriorate its performance. Therefore, it is imperative to analyze the impact of second-, third- and fourth-order dispersion along with first-order dispersion for high data rate optical system.

The complex electric field envelop of CPFSK transmitter using single mode laser diode at the fiber input is given as [1, 8, 10]:

\[ E(t) = E_{\text{in}}(t)e^{j\phi(t)} \]  

(1)

where \( E_{\text{in}}(t) \) and \( \omega_0 \) are the slowly varying complex amplitude and the mean optical frequency respectively. \( E_{\text{in}}(t) \) is further represented as [16-18]:

\[ E_{\text{in}}(t) = \sqrt{2P_t(t)}e^{j\phi_\text{in}(t)} \]  

(2)

where \( P_t(t) \) and \( \phi_\text{in}(t) \) are, respectively, the average transmitted power and angle modulation. Here,

\[ \phi_\text{in}(t) = 2\pi a_k \frac{T}{\tau} \sum_{t=0}^{\infty} p(t-kT)dt + \phi_\text{in}(t) \]  

(3)

with \( a_k \) is the \( k \)th information bit, \( p(t) \) is a unit rectangular pulse of duration \( T \) seconds (bit rate \( B=1/T \) ) and \( \phi_\text{in}(t) \) is the instantaneous phase noise of the transmitting laser. Also, \( h \) and \( f_d \) are the modulation index and frequency deviation respectively related as \( h = 2 f_d T \).

Baveja et al. described the signal propagation over an optical fiber by \( e^{-jBt} \) with \( L \) as the fiber length and \( B \) the propagation constant by [5] :

\[ E_{\text{out}}(\omega) = E_{\text{in}}(\omega)e^{-jBt} \]  

(4)

Since the signal noise or distortion is introduced by the chromatic dispersion rather than fiber loss, therefore the fiber losses can be neglected here [8, 10]. The Taylor series expansion of \( B \) around \( \omega = \omega_0 \) is given as [1, 10]:

\[ B = \beta_0 + (\omega - \omega_0) \frac{\partial B}{\partial \omega} + \frac{1}{2} (\omega - \omega_0)^2 \frac{\partial^2 B}{\partial \omega^2} + \frac{1}{6} (\omega - \omega_0)^3 \frac{\partial^3 B}{\partial \omega^3} + \frac{1}{24} (\omega - \omega_0)^4 \frac{\partial^4 B}{\partial \omega^4} + \cdots \]  

(5)

where \( \beta_0 \) is the group delay per unit fiber length

\[ \frac{d\tau}{d\omega} = -\frac{\lambda^2}{2\pi c} \]  

(6)

is first-order dispersion (1OD)

\[ \frac{d^2\tau}{d\omega^2} = \frac{\lambda^2}{(2\pi c)^2} \left[ \frac{\lambda^2}{\lambda^2} + 2\lambda \frac{\partial \tau}{\partial \lambda} + 6\lambda \frac{\partial^3 \tau}{\partial \lambda^3} \right] \]  

(7)

is second-order dispersion (2OD)

\[ \frac{d^3\tau}{d\omega^3} = -\frac{\lambda^3}{(2\pi c)^3} \left[ \lambda^3 \frac{\partial^3 \tau}{\partial \lambda^3} + 6\lambda \frac{\partial^3 \tau}{\partial \lambda^3} + 6\lambda \frac{\partial^3 \tau}{\partial \lambda^3} + 6\lambda \frac{\partial^3 \tau}{\partial \lambda^3} \right] \]  

(8)

is third-order dispersion (3OD) and

\[ \frac{d^4\tau}{d\omega^4} = \frac{\lambda^4}{(2\pi c)^4} \left[ \lambda^4 \frac{\partial^4 \tau}{\partial \lambda^4} + 12\lambda^2 \frac{\partial^4 \tau}{\partial \lambda^4} + 36\lambda^2 \frac{\partial^4 \tau}{\partial \lambda^4} + 24\lambda \frac{\partial^4 \tau}{\partial \lambda^4} \right] \]  

(9)

is fourth-order dispersion(4OD).

Using eq. (5) in the exponential term of eq. (4), the following expression is obtained:

\[ e^{-jBt} = e^{-jBt} \]  

(10)

Here, the absolute phase \( \phi = B T \) at \( \omega = \omega_0 \) and group delay \( \tau \) corresponding to \( F_1 \) term can be neglected as they produce phase delay of the carrier signal only and doesn’t have any impact on the signal distortion. The first-, second-, third- and fourth-order dispersion parameters in terms of \( F_2, F_3, F_4, F_5 \) respectively are defined as follows:

\[ F_2 = \frac{L}{2} \frac{d^2\tau}{d\omega^2} = \frac{L}{2} \frac{\lambda^2}{2\pi c} \]  

(11)

\[ F_3 = \frac{L}{6} \frac{d^3\tau}{d\omega^3} = \frac{L}{6} \frac{\lambda^3}{(2\pi c)^3} \left[ \lambda^3 \frac{\partial^3 \tau}{\partial \lambda^3} + 6\lambda \frac{\partial^3 \tau}{\partial \lambda^3} + 12\lambda \frac{\partial^3 \tau}{\partial \lambda^3} \right] \]  

(12)

\[ F_4 = \frac{L}{24} \frac{d^4\tau}{d\omega^4} = \frac{L}{24} \frac{\lambda^4}{(2\pi c)^4} \left[ \lambda^4 \frac{\partial^4 \tau}{\partial \lambda^4} + 12\lambda^2 \frac{\partial^4 \tau}{\partial \lambda^4} + 36\lambda^2 \frac{\partial^4 \tau}{\partial \lambda^4} + 24\lambda \frac{\partial^4 \tau}{\partial \lambda^4} \right] \]  

(13)

\[ F_5 = \frac{L}{120d\omega^5} = \frac{L}{120} \frac{\lambda^5}{(2\pi c)^5} \left[ \lambda^5 \frac{\partial^5 \tau}{\partial \lambda^5} + 12\lambda^3 \frac{\partial^5 \tau}{\partial \lambda^5} + 36\lambda^3 \frac{\partial^5 \tau}{\partial \lambda^5} + 24\lambda \frac{\partial^5 \tau}{\partial \lambda^5} \right] \]  

(14)

By substituting eqs. (11), (12), (13) and (14) in eq. (10) and then in eq. (4), we obtain the Fourier domain equivalent as:

\[ E_{\text{out}}(\omega) = E_{\text{in}}(\omega)e^{jF_1(\omega-\omega_0)^4 - jF_2(\omega-\omega_0)^6 + jF_3(\omega-\omega_0)^8 - jF_4(\omega-\omega_0)^{10} + jF_5(\omega-\omega_0)^{12} + \cdots} \]  

(15)

In time domain,

\[ E_{\text{out}}(t) = e^{j\omega_0 t} e^{-jF_2(\omega_0)^2} e^{jF_3(\omega_0)^4} e^{-jF_4(\omega_0)^6} e^{jF_5(\omega_0)^8} \cdots \sqrt{2P_t(t)}e^{j\phi(t)} \]  

(16)
Since $j\omega = \frac{\partial}{\partial t} (j\omega)^2 = -\omega^2 = \frac{\partial^2}{\partial t^2}$ and $(j\omega)^3 = -j\omega^3 = \frac{\partial^3}{\partial t^3}$ etc.

For small signal analysis, assume
\[ E_{\text{out}} (t) = E_{\text{in}} (t) + \delta E (t) \tag{17} \]

where, $\delta E (t)$ is very small and
\[ |\delta E (t)| \ll |E_{\text{in}} (t)| \tag{18} \]

From equation (16) and (17)
\[ \delta E (t) = \left( e^{-jFz^2 \frac{\partial^2}{\partial t^2} + jFz^2 \frac{\partial^2}{\partial t^2} + \ldots} - 1 \right) \sqrt{2P_I(t)} e^{j\phi_0(t)} \tag{19} \]

The field intensity at the fiber output is given as [8]:
\[ P_{\text{out}} (t) = |E_{\text{in}} (t) + \delta E (t)|^2 \approx |E_{\text{in}} (t)|^2 + 2\Re \left[ E_{\text{in}} (t) \cdot \delta E (t) \right] \tag{20} \]

Putting eqs. (2) and (19) in eq. (20) we get
\[ P_{\text{out}} (t) = 2P_I(t) + 2\Re \left[ \left( e^{-jFz^2 \frac{\partial^2}{\partial t^2} + jFz^2 \frac{\partial^2}{\partial t^2} + \ldots} - 1 \right) \sqrt{2P_I(t)} e^{j\phi_0(t)} \right] \ldots \tag{21} \]

\[ \phi_{\text{out}} (t) = \phi_{\text{in}} (t) + \Re \left[ \frac{\delta E (t)}{E_{\text{in}} (t)} \right] \tag{22} \]

\[ \phi_{\text{out}} (t) = \phi_{\text{in}} (t) + 3 \frac{\left( e^{-jFz^2 \frac{\partial^2}{\partial t^2} + jFz^2 \frac{\partial^2}{\partial t^2} + \ldots} - 1 \right) \sqrt{2P_I(t)} e^{j\phi_0(t)}}{\sqrt{2P_I(t)} e^{j\phi_0(t)}} \tag{23} \]

Equations (21) and (23) derived above are giving the optical power and phase at the output of a dispersive optical link. These equations are applicable to any input signal provided that dispersion induced field amplitude $\delta E (t)$ is very small in comparison to the input field $E_{\text{in}} (t)$ i.e. $|\delta E (t)| \ll |E_{\text{in}} (t)|$.

The impulse response of the system is calculated by taking the inverse Fourier transform ($F^{-1}$) defined by $h(t) = F^{-1} \{ |\exp(jH(\omega))| \}$ where $F$ represents the Fourier transform. For an angle modulated signal propagating in an optical fiber, the nonlinear SPM effect arises due to GVD induced PM-AM conversion [13]. The Fourier transform of the output phase $\phi_{\text{out}} (\omega)$ and the output power fluctuation $\xi_{\text{out}} (\omega)$, under linear phase approximation can be expressed in terms of the corresponding input signal $\phi_{\text{in}} (\omega)$ and $\xi_{\text{in}} (\omega)$ through the transfer function matrix [13, 17]:

\[
\begin{bmatrix}
\phi_{\text{out}} (\omega) \\
\xi_{\text{out}} (\omega)
\end{bmatrix} = D_{\text{SPM}}(z, \omega)
\begin{bmatrix}
\phi_{\text{in}} (\omega) \\
\xi_{\text{in}} (\omega)
\end{bmatrix}
\tag{24}
\]

where $\xi_{\text{out}} (\omega)$ and $\phi_{\text{out}} (\omega)$ are the Fourier transforms of $\delta E (t)$ and $\phi (t)$ respectively and if $\langle S \rangle = P_z^2$, then $D_{\text{SPM}}(z, \omega)$ the system transfer function described as [13]:

\[
D_{\text{SPM}}(z, \omega) = \frac{\cos (F(\omega))}{2\langle S \rangle} - \frac{\sin (F(\omega))}{\cos (F(\omega))} \tag{25}
\]

For dispersion limited fiber link, neglecting the absolute phase and group delay corresponding to $F_1$ term as these terms generate only phase delay of the carrier signal and doesn’t distort the signal, so

\[
F(\omega) = \left( \frac{1}{2} \beta_2 \omega^2 + \frac{1}{6} \beta_3 \omega^3 + \frac{1}{24} \beta_4 \omega^4 + \frac{1}{120} \beta_5 \omega^5 + \ldots \right) \tag{26}
\]

where $z$ is the fiber length. Equation (25) establishes the relationship between the input vector $\xi_{\text{in}} (\omega) = [\phi_{\text{in}} (\omega), \xi_{\text{in}} (\omega)]$ and the output vector $\xi_{\text{out}} (\omega) = [\phi_{\text{out}} (\omega), \xi_{\text{out}} (\omega)]$ in a dispersion limited fiber link.

To compute the relative intensity noise (RIN) spectrum at the fiber input we need to know the Fourier components of the power and phase fluctuation at the fiber input as well as their correlation properties. The single sided RIN spectrum at the fiber output is given by [12]:

\[
\text{RIN}(\omega) = \frac{2\langle |\tilde{P}_N(z, \omega)|^2 \rangle}{P^2(0)} \tag{27}
\]

Here $P_N(z, \omega)$ is the normalized power defined as $P(z, \omega) = p_N(z, \omega) e^{-az}$ and $\tilde{P}_N(z, \omega)$ is the Fourier transform of $p_N(z, \omega).\tilde{P}_N(z, \omega)$ is the amplitude of the Fourier component of the power fluctuation at the fiber output and $P(0)$ is the average power at the fiber input.

The BER of an optical heterodyne direct detection NRZ-CPFSK receiver with delay-and-multiply discriminator having $\omega_d = 2\pi f_d$ as the peak frequency deviation and $h = 2\pi / T$ as the modulation index utilizing Guass Quadrature rule can be evaluated by [11, 13]:

\[
\text{BER} = \int \frac{P(\epsilon | x) \cdot \rho(\epsilon) d(\epsilon)}{\int P(\epsilon | x) \cdot \rho(\epsilon) d(\epsilon)} \tag{28}
\]

where

\[
P(\epsilon | x) = \frac{1}{2} \left[ 1 - \frac{1}{2} \sum_{2n+1} \left( -\frac{1}{2} \right)^n K_n \left( \frac{x}{2} \right)^n + K_{n+\frac{1}{2}} \left( \frac{x}{2} \right)^{n+\frac{1}{2}} \right] \cos [2n + 1] (\Delta \xi + \xi) \tag{29}
\]

and $\rho$ is the intermediate frequency signal to noise ratio (IF SNR), $K_n(x)$ is the $n^{th}$ order modified Bessel’s function of first kind.
3. RESULTS AND DISCUSSIONS

As per ITU-T Recommendations, G.653 [15] DS fiber is assumed near 1550nm up to fourth-order dispersion term,
\[ \frac{d^4 \tau}{d \lambda^4} = (\lambda - \lambda_0)^4 S_0 \]
where \( S_0 \) is zero dispersion slope defined as \( S_0 = \frac{d^2 \tau}{d \lambda^2} \) with value 0.085 ps/nm²/km and zero dispersion wavelength \( \lambda_0 = 1550nm \). Also, \( \frac{d^2 \tau}{d \lambda^2} = 5 \times 10^{-3} \) ps/nm/km,
\[ \frac{d^3 \tau}{d \omega^3} = 0.138 \text{ ps}^3/\text{km} \] and \( \frac{d^3 \tau}{d \omega^3} = 0.000618 \text{ ps}^3/\text{km} \), attenuation constant \( \alpha = 0.25 \text{ dB/km} \) and fiber length \( z = 1200 \text{ km} \). The parameters considered for CW laser are relaxation oscillation frequency \( f_R = 20.25 \text{ GHz} \), damping rate \( \gamma = 63.29 \text{ GHz} \), photon life time \( \tau_{\text{ph}} = 118 \text{ ps} \), average photon density \( \langle n \rangle = 4.5 \times 10^5 \) and maximum modulation width \( f_{\text{max}} = 63.47 \text{ GHz} \). The power transfer function for the system has been plotted in Figure 1 using equation (25) for modulation frequencies varying from 0 to 25GHz. Here first-, second-, third and fourth-order dispersion terms and their combinations with different FWHM linewidth of the laser source are considered.

Figure 1(a) shows the plot with 1OD term \( F_2 \). Since the power transfer function has higher value for high modulation frequency, the frequency response with \( F_2 \) only is better at high modulation frequencies and smaller linewidths. For 2OD term \( F_3 \), the response is shown in Figure 1(b). It is seen that the variation of power with respect to modulation frequency is very small as compared to \( F_2 \); although the trend is similar to that of \( F_2 \) i.e the performance of the system is better for smaller linewidths. Individually, the effect of 3OD term \( F_4 \) is negligible. Figure 1(c) shows the combined effect of \( F_2 \), \( F_3 \) and \( F_4 \) on the frequency response. It is seen that when combination of \( F_2 \), \( F_3 \) and \( F_4 \) is taken, the spectral characteristics depend on linewidth of the laser source and the output is better for narrow linewidths. Also, it is apparent from the Figure 1 that 2OD term indeed affects the frequency response at higher modulation frequencies even if 1OD is zero.

Figure 2 shows the spectra of the system for varying input powers in terms of relative intensity noise (RIN). Using the estimated laser intrinsic parameters, RIN at the fiber output has been computed by considering the laser
noise alone. Here, the increase in the input power of the laser source from 4 to 8dBm leads to an increase in RIN by about 5dB and increase in the dip frequency of about 2GHz. It is found that the relative intensity noise increases as the input power increases thus further increasing the power required to compensate the noise.

Figure 3 shows the comparison of bit error rate (BER) versus the IF SNR curves obtained for first and second-order dispersion parameter for heterodyne CPFSK system by varying the linewidth of the source from 2MHz to 10MHz. It can be clearly seen that there is a significant decline in the BER value with the reduction in the linewidth of the laser.

It is seen in the Figure 3(a) that the BER for the system with F2 only falls from 5×10⁻³ to 10⁻⁸ for IF SNR=10dB if the value of FWHM linewidth decreases from 10MHz to 2MHz. Whereas Figure 3(b) shows that the system performs better for second-order dispersion F3 only as the BER>10⁻⁹ and declines from 10⁻¹⁵ to 10⁻¹⁹ for IF SNR=10dB and the same decrease in the linewidth. Also, the BER value exponentially reduces with the increase in the IF SNR. The results show that the BER value reduces considerably for a CPFSK system if second-order dispersion F3 only is considered as compared to the system if first-order dispersion F2 only is taken into account. It is clearly observed from the Figure 2 that the 2OD has minor effect on the BER performance. Hence, the CPFSK system performs better for narrow linewidth.

**CONCLUSION**

The performance of 40Gbps CPFSK system with optical heterodyne receiver is analyzed by taking into consideration the cumulative effect of group velocity dispersion, higher order dispersion and self-phase modulation. The system performance has been evaluated using single mode dispersion shifted fiber with 1550nm zero dispersion wavelength for various values of the input power, linewidth of the source and chromatic dispersion. The CPFSK system performance is severely degraded due to chromatic dispersion and self-phase modulation. It is found that the 2OD and 3OD has minor effect on the frequency response and BER performance as compared to first order dispersion. Also, the spectral characteristics and BER performance are better for narrow linewidth of the source. The nonlinear SPM effect in conjunction with chromatic dispersion results in additional phase shift thus restricting the maximum allowable launched input power into the fiber. It is found that the relative intensity noise increases as the input power increases thus increasing the power penalty. It is evident that the CPFSK system performs better for narrow linewidth of the source and for smaller input powers of the source.

**REFERENCES**


AUTHOR PROFILE

1. Anu Sheetal receivd her Bachelor’s degree in Electronics and Instrumentation Engineering (with Honors) from the Department of Electronics & Communication Engineering, Punjabi University, Patiala, India in 1994 and Master’s degree in Electronics Engineering from Punjab Technical University, Jalandhar, India in 2003. She obtained his Ph.D. degree from Punjab Technical University, Jalandhar, in 2012. She worked as a Design Engineer at Gilard Electronics Private Limited, Mohali, from 1994 to 1997. In 2004, she joined Guru Nanak Dev University, Regional campus, Gurdaspur, Punjab, India in the Department of Electronics and Communication Engineering as a lecturer and became Assistant Professor in the Department of Electronics and Communication Engineering. Presently, she is working as Sr. Assistant Professor and Incharge of the department in the same institute. Her present interests are Optical Communication Systems, soliton transmission and DWDM Networks, WDM-PON, RoF etc. She has over 50 research papers published/preseated in reputed International/National Journals/Conferences to her credit. She is a life member of The Institution of Electronics and Telecommunication Engineers (IETE), New Delhi (India), Indian Society of Technical Education (ISTE), Optical Society of India, Kolkata (OSI), International Association of Engineers (IAENG). She is acting as technical reviewer for many International Journals of Elsevier, SPIE and Taylor & Francis. She has organized two faculty development programs (FDPs) and three national level Seminars/Conferences and is also frequently delivering lectures/expert talk in various Short term courses/FDPs etc.