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Non-Linear Modeling and Control of Quadrotor

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ABSTRACT

Quadrotor, one of the most popular form of civil UAV which exists in various sizes and prices. A mathematical model and control of such a UAV is described in this paper. It introduces a mathematical model obtained by using Newton-Euler equations. A six degrees of freedom model is elaborated starting from the general equation until the state-space form, along with linearization assumptions. The controller part consist of obtaining the best control strategy to stabilize the system based on PID control. The full dynamic model of the quadrotor is considered for designing the controllers and the simulations are done and analyzed in MATLAB-SIMULINK.

Keywords:*Feedback,linearization,.PIDcontrol,Quadrotor*
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1. INTRODUCTION

Quadrotor, also called as a quadcopter is becoming popular in the civil and military applications due to their vertical takeoff and landing (VTOL) capability and safer to interact. These type of vehicles is being used in various areas including surveillance, border patrol, disaster management, photography etc. For these advantages, quadrotors have received much more interest in UAV research. It is a four propeller helicopter. The basic motions of this type of vehicles are generated by varying the rotor velocities of all four rotors, thus by changing the thrust moments. The quadrotor tilts towards the direction of low lift rotor, which accelerates along that direction. Spinning directions of the rotors are set to balance the moments. This is also used to produce the desired yaw motions.

The quadrotor considered here, is an under-actuated system with only four inputs and six outputs. Hence the states are highly coupled. To deal with such a system, many modeling approaches have been presented and various control methods are proposed. The Proportional- Integral-Derivative (PID) controller is used for the attitude control of a quadrotor which is dealt with [1]. The idea of an independent control action for the variables of the attitude control is shown. A similar decoupled control law is used in [2], here the gyroscopic effects are neglected and rotor dynamics are included then designed the PID controllers. Then a Linear Quadratic Regulator (LQR) control is compared with, and for the system which is linearized around each of the state to accommodate a wider flight motion. The similar linearization method has been used in [3], in which full control of quadrotor employing the LQR technique, after using model linearization using small angle approximation and then giving the equilibrium conditions. In the nonlinear techniques, feedback linearization control (FBL) is one of the methods mostly seen in literatures. Two methods for the quadrotor control using feedback linearization are dealt in [3]. A similar extended system is shown in [4] where the repeated differentiation is performed and then applied the small angle approximations.

Feedback linearization technique by dynamic inversion for the control of trajectory tracking is given in [5]. Separate laws are used for rotational and translational dynamics after repeated differentiation is used. It is followed by a linear auxiliary control input to stabilize the error dynamics. A similar technique is used for the control of attitude dynamics in [6], with these attitudes are chosen in the outputs. Through small angle approximation for the attitude variables, the matrix to be inverted is obtained directly from the dynamics. The obtaining linearized model can be controllable using any of the standard techniques like PID or back stepping controller.

An analysis between PID controller, inverse control, sliding mode control and back-stepping control is dealt in [7]. Upon the total error criterion for evaluation of the performance, the best controller be proved as the sliding mode controller. A detailed comparison between feedback linearization control and sliding mode control is performed in [4].

Our study presents two controllers for a quadrotor system. The first one is a PID controller for the linearized model the vehicle. A PID controller calculates the difference between a set point and a desired set point in the process as an "error" value. The controller tries to reach the set point by downloading the minimum value of the error. The control output is passed through three separate mathematical operations and is obtained by summing. The second one operates on a feedback linearization (FL) technique for an integrated X-Y- Z control. Feedback linearization controllers can be directly applied to nonlinear dynamics without linear approximations. We simplify the equation of system dynamics for the FL controller in order to avoid complex calculations involving repeated differentiation. Although this controller is simple to implement, model uncertainty can cause performance degradation or instability of the closed loop system, the FL controller is quite sensitive to external disturbance or sensor noise.

Compared to the previously cited works here both the nonlinear and linear model of the quadrotor is considered for control purpose under the feedback linearization and PID control schemes respectively. It can be seen that the linearized model is obtained only through huge approximations of some terms which may affect the performance of the system. Hence a nonlinear controlling scheme which is the feedback linearization control is proposed and its simulation part is considered in our future work.

The rest of the paper is organized as followed: Section 2 presents the dynamics of the quadrotor. Section 3 introduces suitable controllers for assuring a stable and optimal system. Section 4 shows the simulation results on the quadrotor. Finally, Section 5 summarizes findings of work to improve the capabilities of the quadrotor.

2. MATHEMATICAL MODELING

Quadrotor is an under-actuated system because it has six degrees of freedom but only four actual inputs. The six degrees of freedom includes translational motion in three directions (X,Y,Z) and rotational motion around three axes $(\emptyset, \theta, \psi)$. The schematic configuration of a quadrotor is shown in Fig. 1. Four rotors, each of which is driven by a motor are mounted at two orthogonal directions and rotates in a direction as the circular arrow does. In the Fig. 1, F_i is the thrust moment produced by the rotor i. To mathematically illustrate the dynamics of the quadrotor, a co-ordinate system should be defined. The co-ordinate system can be divided into an earth frame {E} and a body frame {B}. The body frame which is represented by {B} with its origin at the center of mass and earth frame which is represented by {E} are as shown in Fig. 1.

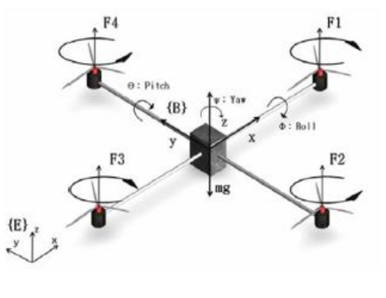

Fig. 1. A quadrotor helicopter configuration

The rotors (1,3) and rotors (2,4), are the two rotor pairs which rotate in opposite directions so that the moment produced by them cancel each other. A roll angle($\emptyset$), along the X-axis of the body frame, can be obtained through increasing the angular velocity of rotor (2) and decreasing that of rotor (4) while keeping the total thrust constant. Alike increasing the angular velocity of rotor (3) and decreasing that of rotor (1) to produce a pitch angle (θ), along the Y -axis of the body frame. In order to perform yawing motion(ψ), along the Z-axis of the body frame, the angular velocity of (1,3) are increased and that of (2,4) are decreased.

The equations describing the dynamics of a quadrotor are basically those of a rotating rigid body which can be derived with Newton-Euler formalism. The complicated motions of a quadrotor can be described by two typical groups of equations each of which represents a subsystem with coupled terms. The first group is related to the translational positions and the second group related to the rotational angles.

Deriving mathematical modeling or differential equations is necessary for the control of the quad-rotor position (X,Y), altitude (Z), attitude (($\emptyset, \theta$) and heading(ψ). However, it is hard for the complicated structure of the quadrotor to express Fig. 1. A quadrotor helicopter configuration [8] its motion with only a simple modeling. In addition, since the quadrotor UAV includes highly non-linear factors, we need to consider several assumptions in order to get a desired model

1) The body is rigid and symmetrical.
2) The rotors are rigid, i.e. no blade flapping occurs.
3) The difference of gravity by altitude or the spin of the Earth is minor.
4) The center of mass and body frame origin coincide.

These assumptions can be formed because of slower speed and lower altitude of the quadrotor UAV as compared to a regular aircraft. Under these assumptions, it is possible to describe the fuselage dynamics. As already mentioned, to mathematically illustrate the fuselage dynamics of the quadrotor UAV, a co-ordinate system should be needed.

The rotational transformation matrix between the earth frame and the body frame can be obtained based on Euler angles (($\emptyset, \theta, \psi$). These are three angle introduced to describe the orientation of a rigid body. To describe such an orientation in the 3-D Euclidian space, three parameters are required. They can be given in several ways, here ZYX Euler angles are used. The Euler angles represent a sequence of three elemental rotations. i.e., rotations about the axes of a co-ordinate system. The orientation combination used is described by the following rotation matrices:

$$R_X(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix},$$

$$R_Y(\theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix},$$

$$R_Z(\psi) = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where c and s indicate the trigonometric cosine and sine functions respectively. So the rotational transformation matrix between the earth frame and the body frame is given by the following equation:

$$R_{ZYX}(\phi, \theta, \psi) = R_Z(\psi) R_Y(\theta) R_X(\phi), \qquad (1)$$

Where,

$$R_{ZYX} = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (2)$$

Thus here we obtained the rotational matrix R_{ZYX} that converts between body frame and earth frame. Now we give the non-linear model of a quadrotor.

A. Non-linear Model

The goal of this section is to obtain a deeper understanding of the dynamics of the quadrotor and to provide a model that is sufficiently reliable for simulating and controlling its behavior. Let us call $[X\ Y\ Z\ \emptyset\ \theta\ \psi]^T$, the vector containing the linear and angular position of the quadrotor in the earth frame and $[u\ v\ w\ p\ q\ r]^T$, the vector containing linear and angular velocities in the two frames. The transformation of velocities between the earth frame and body frame can be derived from,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = R_{ZYX} \begin{bmatrix} u_B \\ v_B \\ w_B \end{bmatrix}, \quad (3)$$

where u_B; v_B and w_B are the translational velocities in the body frame. Similarly, positions, forces, moments, accelerations and rotational velocities can be transformed based on R_{ZYX} between co-ordinate systems. In the body frame, the forces are presented as,

$$F_B = \begin{bmatrix} F_{XB} \\ F_{YB} \\ F_{ZB} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}. \quad (4)$$

where F_i (i = 1; 2; 3; 4) is the thrust moment produced by the rotor i .It is defined by,

$$F_i = b\omega_i^2 ,$$

where b is a positive constant that denotes the thrust factor of propeller and ω_i is the angular velocity of the motor i. Accordingly, in the earth frame, the forces can be defined as,

$$F_E = \begin{bmatrix} F_{XE} \\ F_{YE} \\ F_{ZE} \end{bmatrix} = R_{ZYX} F_B = \begin{bmatrix} c\phi s\theta c\psi + s\phi s\psi \\ c\phi s\theta s\psi - s\phi c\psi \\ c\theta c\phi \end{bmatrix} \sum_{i=1}^{4} b\omega_i^2 \quad (5)$$

Therefore, equations of motion in the earth frame for the

$$m \begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \begin{bmatrix} F_{XE} \\ F_{YE} \\ F_{ZE} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix},$$

translational dynamics (X, Y, Z) are derived by the Newton's laws,

(6)

Where m is the mass of the quadrotor and g the acceleration due to gravity. By extending the Newton's law on rotational

$$\ddot{\phi} = \frac{lb}{I_X}(\omega_4^2 - \omega_2^2),$$
$$\ddot{\theta} = \frac{lb}{I_Y}(\omega_3^2 - \omega_1^2),$$
$$\ddot{\psi} = \frac{d}{I_Z}(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2),$$

dynamics$(\emptyset, \theta, \psi)$, its equation can be written as,

(7)

Where l is the distance from the center of rotors to the quadrotor center of mass. d indicate drag coefficient.I_X; I_Y ; I_Z are moments of inertia of the quadrotor. As a consequence, the complete dynamic model which governs the quadrotor is as follows:

$$\ddot{X} = (s\phi s\psi + c\phi s\theta c\psi)\frac{u_1}{m},$$
$$\ddot{Y} = (c\phi s\theta s\psi - s\phi c\psi)\frac{u_1}{m}, \quad (8)$$
$$\ddot{Z} = (c\phi c\theta)\frac{u_1}{m} - g,$$

$$\ddot{\phi} = \frac{l}{I_X}u_2,$$
$$\ddot{\theta} = \frac{l}{I_Y}u_3, \quad (9)$$
$$\ddot{\psi} = \frac{1}{I_Z}u_4,$$

where u_i(i = 1; 2; 3; 4) are control inputs of the model,

$$\begin{aligned} u_1 &= b(F_1 + F_2 + F_3 + F_4), \\ u_2 &= b(F_4 - F_2), \\ u_3 &= b(F_3 - F_1), \\ u_4 &= d(F_1 + F_3 - F_2 - F_4). \end{aligned} \quad (10)$$

Equation (10) describes the thrust moments acting on quadrotor as shown in Fig. 1.The total thrust developed by the four rotors is given u_1, the rolling and pitching moments occur due to the difference in thrust produced by the opposing rotors is given by u_2 and u_3. Yawing moment is caused by the drag force acting on all the propellers and opposing their rotation which is given by u_4. By using (8) and (9), a compact non-linear model of the quadrotor is given as,

$$\dot{\mathbf{X}} = f(\mathbf{X}, U) = f(\mathbf{X}) + g(\mathbf{X})U$$

Where $X = [XY\,Z\,\emptyset\,\theta\,\psi\;\dot{X}\,\dot{Y}\,\dot{Z}\;\dot{\emptyset}\;\dot{\theta}\;\dot{\boldsymbol{\psi}}]^T$ .are the state variables, f(X) and g(X) are smooth functions on X.
Equivalently using $\bar{X} = [X\;Y\;Z\;\emptyset\;\theta\;\psi]^T$ and $= [u_1 u_2\,u_3\,u_4]^T$, in the vector form as,

$$\ddot{\bar{\mathbf{X}}} = \hat{f}(\bar{\mathbf{X}}) + \hat{g}(\bar{\mathbf{X}})U, \quad (11)$$

$$\hat{f}(\bar{\mathbf{X}}) = \begin{bmatrix} 0 \\ 0 \\ -g \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\hat{g}(\bar{\mathbf{X}}) = \begin{bmatrix} \frac{s\phi s\psi + c\phi s\theta c\psi}{m} & 0 & 0 & 0 \\ \frac{c\phi s\theta s\psi - s\phi c\psi}{m} & 0 & 0 & 0 \\ \frac{c\phi c\theta}{m} & 0 & 0 & 0 \\ 0 & \frac{l}{I_X} & 0 & 0 \\ 0 & 0 & \frac{l}{I_Y} & 0 \\ 0 & 0 & 0 & \frac{1}{I_Z} \end{bmatrix}.$$

Hence we obtained the non-linear model of a quadrotor as the above equations. The linearized model of the quadrotor is discussing in the next part.

B. Linearized Model

This section deals with the linearized model of the quadrotor. Set U the control vector, $U = [u_1 u_2\,u_3\,u_4]^T$and defining the state vectors as, $X = [XY\,Z\,\emptyset\,\theta\,\psi\;\dot{X}\,\dot{Y}\,\dot{Z}\;\dot{\emptyset}\;\dot{\theta}\;\dot{\boldsymbol{\psi}}]^T$.
The linearizations procedure is developed around an equilibrium point $\bar{X}$, which for a fixed input $\bar{U}$ is the solution of the algebraic system: or rather that value of states vector, which on fixed constant input is the solution of algebraic system:

$$f(\bar{\mathbf{X}}, \bar{U}) = 0 \ ,$$

since the function f is nonlinear, problems related to the existence an uniqueness of the solution of system arise. In particular, for the system in hand, the solution is difficult to find in closed form because of trigonometric functions related each other in no-elementary way. For this reason, the linearization is performed on a simplified model called to small oscillations. This simplification is made by approximating the sine function with its argument and the cosine function with unity. The approximation is valid if the argument is small. The resulting system can be as,

$$\dot{\hat{\mathbf{X}}} = \hat{f}(\mathbf{X}, U).$$

Inorder to perform the linearization, an equilibrium point is needed. Such an equilibrium point can be,

$$\bar{\mathbf{X}} = [0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0]^T \ ,$$

from the equations (8) and (9) we can find that the equilibrium point is obtained by constant input value,

$$\bar{U} = [mg\;0\;0\;0]^T,$$

After determining the equilibrium point and the corresponding nominal input, we have that the matrices associated to the linear system are given by the relations:

$$\begin{aligned} \dot{\hat{\mathbf{X}}} &= A\mathbf{X} + BU, \\ Y &= C\mathbf{X}, \end{aligned} \quad (12)$$

$$A = \frac{\partial f(\mathbf{X}, U)}{\partial \mathbf{X}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad (13)$$

$$B = \frac{\partial f(\mathbf{X}, U)}{\partial U} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1/I_X & 0 & 0 \\ 0 & 0 & 1/I_Y & 0 \\ 0 & 0 & 0 & 1/I_Z \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (14)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

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Thus we obtained the linearized model of the quadrotor as the above equation. The next section deals with the control part of the quadrotor.

3. CONTROL DESIGN

This section discuss with the control part of the quadrotor. A PID controller for the linearized model is discussing and also the feedback linearization control theory is included for the non-linear model.

A. PID Control for the linearized model

PID is a control mechanism used in common industrial control systems. It is also widely used in quadrotor control. A PID controller calculates the difference between a set point and a desired set point in the process as an "error" value. The controller tries to reach the set point by downloading the minimum value of the error. The control output is passed through three separate mathematical operations and is obtained by summing. System effects are as follows. Proportional Effect (P): Effective as the output multiplied by a certain "gain" value of the error, calculates the current error. Integral Effect (I): The effect of the control is proportional to the sum of all the errors in the moment up to the moment the effect is calculated. In other words, the integral effect means the sum of errors the system has made in the past. Derivative Effect (D): It has a proportional effect on the output of the system, according to the change of the error. So it calculates the prediction of the future error. Karl Arstom defines this algorithm which has a wide application area as follows:

$$u(t) = K_p e(t) + K_i \int_0^t e(v)d(v) + K_d \frac{de(t)}{d(t)}, \qquad (16)$$

Where, K_p proportional coefficient, K_i integral coefficient and K_d is the derivative coefficient.
The PID controller block diagram is shown in Fig. 2.

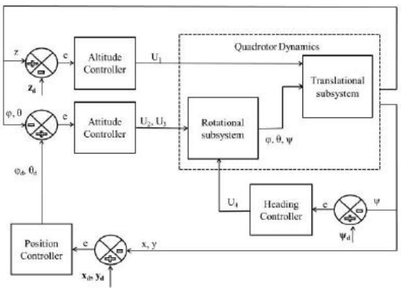

Fig. 2. PID controller block diagram - Quadrotor complete system

The PID controller for the quadrotor is developed based on the fast response. One aspect of the controller selection depends on the method of control of the quadrotor. It can be mode based or non-mode based. For the mode based controller, independent controllers for each state are needed, and a higher level controller decides how these interact. On the other hand for a non-mode based controller, a single controller controls all of the states together. However the adopted control strategy is summarized in the control of two subsystems; the first relates to the position control while the second is that of the attitude control. The quadrotor model above can be divided into two subsystems: A fully-actuated subsystem S1 that provides the dynamics of the vertical position (Z) and the yaw angle (ψ),

$$\begin{bmatrix} \ddot{Z} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} (cos\phi cos\theta)\frac{u_1}{m} - g \\ u_4 \end{bmatrix}. \qquad (17)$$

A subsystem S2 representing the under-actuated subsystem which gives the dynamic relation of the horizontal positions (X; Y) with the pitch and roll angles ($\emptyset, \theta\_$) ,

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} = \begin{bmatrix} u_1 cos\phi & u_1 sin\phi \\ u_1 sin\phi & -u_1 cos\phi \end{bmatrix} \begin{bmatrix} sin\theta cos\psi \\ sin\psi \end{bmatrix}, \qquad (18)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u_2 l \\ u_3 l \end{bmatrix}. \qquad (19)$$

The PID control is applied to the equations above with inputs u1; u2; u3; u4 and outputs $\emptyset, \theta, \psi$ and X, Y,Z. Though these methods were rather successful in local analysis of nonlinear systems affine in control they usually fail to work for a global analysis. For the fully-actuated subsystem we can construct a rate bounded PID controllers to move states X; Y;Z and $\emptyset, \theta, \psi$ to their desired values. Next is a nonlinear technique used for quadrotor control called feedback linearization.

B. Feedback linearization control for the nonlinear model

The technique is based on the construction of a nonlinear inverse dynamic controller for a system of the form:

$$\tilde{\dot{\mathbf{X}}} = f(\mathbf{X}) + g(\mathbf{X})U, \qquad (20)$$
$$\tilde{Y} = h(\mathbf{X}),$$

where f(X) and g(X) are vector fields in R^n , U is the input and Y is the output. The control design process is to find an integer ρ and a state feedback,

$$U = \alpha(\mathbf{X}) + \beta(\mathbf{X})V, \qquad (21)$$

where V is a new control variable, $\propto$ and β are smooth functions defined in a neighborhood of some point $X_0 = R^n$ such that the closed loop system has the property that the ρth order derivative of the output is given by

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$$Y^{\rho} = V, t = \Gamma, \qquad (22)$$

where Γ is an open interval containing t = 0. This problem is termed as (local) input-output feedback linearization. The point around which the linearization is performed is called the analysis point.

The quadrotor under consideration is an under-actuated system, and g(X) in (20) is not invertible. So the non-linear terms in (20) cannot be directly canceled by inverting g(X). To make this system feedback linearizable, one may consider choosing $\emptyset, \theta, \psi$ and Z as output variables. If X; Y;Z and are chosen as output variables, it can be easily seen that u2 and u3 do not appear in (8) and last equation of (9), so we need to differentiate these equations until the input terms appear. Because of the repeated differentiation, the FL controller design involves complex computation and several derivative terms that are quite sensitive to noise. In order to reduce the number of complicated derivative terms involved in further differentiations of X and Y , we first approximate (8) and last equation of (9) using the small angle assumptions. The quadrotor output dynamics can be placed in state space form as follows:

$$\ddot{\tilde{\mathbf{X}}} = \begin{bmatrix} 0 \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} s\phi s\psi + c\phi s\theta c\psi & 0 & 0 & 0 \\ c\phi s\theta s\psi - s\phi c\psi & 0 & 0 & 0 \\ c\phi c\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} U, \qquad (23)$$

note that the matrix multiplying the control u_1, u_2, u_3, u_4 is singular which implies that there is no static state feedback that will linearize. In this case we must use dynamic inversion and this can be achieved by dynamic extension or simply by placing two integrator before ul input. Thus differentiating equation (8) two more times we obtain the following output dynamics for the quadrotor

$$\begin{bmatrix} X^{(4)} \\ Y^{(4)} \\ Z^{(4)} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} f_X \\ f_Y \\ f_Z \\ 0 \end{bmatrix} + \begin{bmatrix} g_{X\ddot{u}_1} & g_{Xu_2} & g_{Xu_3} & g_{Xu_4} \\ g_{Y\ddot{u}_1} & g_{Yu_2} & g_{Yu_3} & g_{Yu_4} \\ g_{Z\ddot{u}_1} & g_{Zu_2} & g_{Zu_3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \qquad (24)$$

where f's and g's are complicated nonlinear functions of state and their derivatives. The dynamic state feedback law that linearize and decouple the quadrotor outputs can be calculated as follows:

$$\begin{bmatrix} \ddot{u}_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} g_{X\ddot{u}_1} & g_{Xu_2} & g_{Xu_3} & g_{Xu_4} \\ g_{Y\ddot{u}_1} & g_{Yu_2} & g_{Yu_3} & g_{Yu_4} \\ g_{Z\ddot{u}_1} & g_{Zu_2} & g_{Zu_3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} V_1 - f_X \\ V_2 - f_Y \\ V_3 - f_Z \\ V_4 \end{bmatrix} \qquad (25)$$

where V_1; V_2; V_3 and V_4 are new control inputs such that resulting closed loop system is in the form:

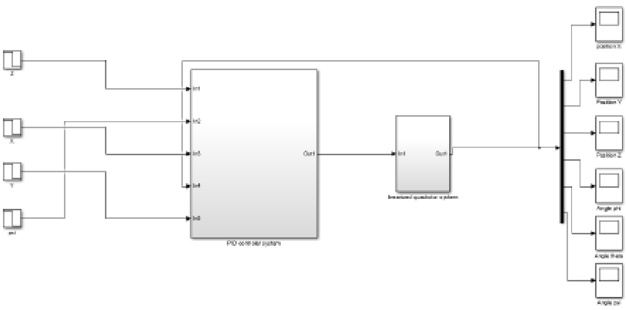
$$\begin{bmatrix} X^{(4)} \\ Y^{(4)} \\ Z^{(4)} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \qquad (26)$$

The control law fully linearize the quadrotor dynamics described by equations and leave no unobservable zero dynamics. The inverse of the decoupling matrix in is non singular as along as the, $U_1$is nonzero. This fact agrees well with the intuition that no amount of rolling or pitching will effect the motion of the quadrotor aircraft if there is no thrust to effect the acceleration. The simulation part of this control scheme is planning to be included as a part of our future work.

4. SIMULATION RESULTS

The proposed PID controller is tested on the dynamic model developed in the SIMULINK environment and is shown in Fig. 3 .

Fig. 3. PID controller simulink diagram - Quadrotor complete system



The nominal parameters and the initial conditions of the quadrotor for simulation are:

I_1 = I_2 = 1:25Ns^2/rad:

I_3 = 2:5Ns^2/rad:

m = 2kg:

l = 0:2m:

g = 9:8ms^2:

The values of the PID controller gains are as shown in Table I,

TABLE I
GAIN VALUES FOR PID CONTROLLER

Controller	K_P	K_I	K_D
X	5	3	3
Y	15	10	10
Z	15	10	10
ϕ	6	1.5	1.75
θ	5	3	3
ψ	6	1.5	1.75

the step response of

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position and attitude of the designed controller are shown in Fig. 4 to Fig. 9. Positions X; Y;Z followed by the roll angle, pitch angle and yaw angle. The desired attitude commands are provided from a block, which contains step functions for each of the variables. The step function starts with an initial condition of 10 deg for the three angles and 5 m for height and falls to 0 deg for the angles and 3 m for the height with a step time of 5 s.

The simulation results show that the PID controllers are able to robustly stabilize the quadrotor helicopter. From Fig. 4 it can be seen that the PID controller step response for position -X which tracks the given value at the step time of 5 s. Similarly Fig. 5 shows the PID controller step response of position -Y , which also tracks the given value of step at the step time of 5 s. From Fig. 6 it can be seen that PID controller step response of the altitude or the position- Z tracks a value of 50 m at the step time of 5 s. That means all the three positions moves to the desired position values while Fig. 7 and Fig. 8 shows that the PID controller step response for the roll and the pitch angles are zero. Also Fig. 9 shows the PID controller step response for the yaw angle. It can be seen that the yaw angle tracks the given value of step at 5 s.

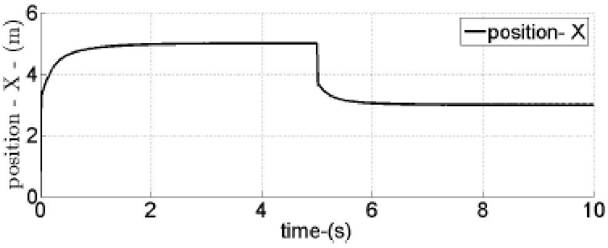


Fig. 4. PID controller step response for position-X

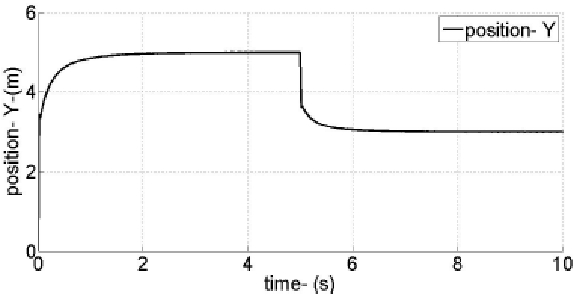
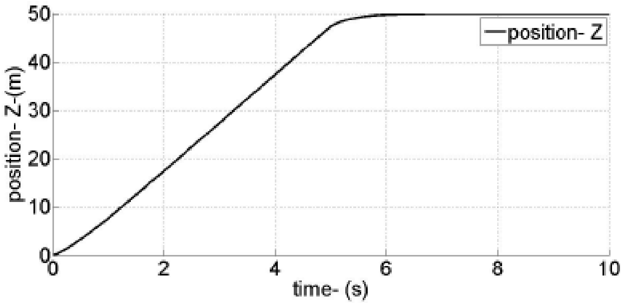


Fig. 6. PID controller step response for position -Z

Fig. 5. PID controller step response for position -Y



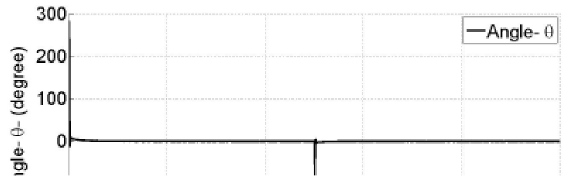


Fig. 7. PID controller step response for angle ∅

time- (s)

-200 0 2 4 6 8 10
time- (s)

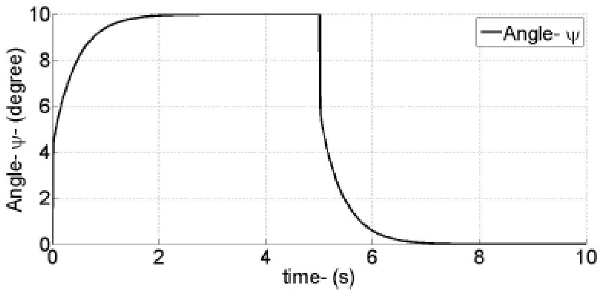


Fig. 9. PID controller step response for angle ψ

Through using the proposed PID controller method strategy, good performance can be shown from the speed of response of the quadrotor.

5. CONCLUSION

In this work the mathematical model of a quadrotor (both linear and non-linear models) was obtained and simulated using two controllers in MATLAB-SIMULINK environment. It is found that the applied PID controller for the linearized model control the system properly and also the feedback linearization controller controls the non-linear model. That means the proposed control schemes has been successfully applied to the fully autonomous quadrotor system.

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