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Performance Analysis of Linearized Controllers for a Nonlinear model of Inverted Pendulum

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ABSTRACT

Inverted pendulum is a fundamental benchmark in control theory and robotics. Numerous practical applications of inverted pendulum renders its study more interesting and vital for real life applications. In robotics, transport machines, large scale constructions, the segway and hoverboard riding and most important application throws light to the operation of rocket propellers, guided misiles and flight controls. The above applications paved the way for inverted pendulum as a system of great importance for engineers. The balancing of inverted pendulum along a vertical position by applying force to the cart is a typical problem in the area of automatic control, which is usually termed as Pole-Cart platform. Its popularity derives from the fact that the system is unstable without proper control. Therefore, it is of vital importance to adopt a proper and efficient control strategy to stabilize inverted pendulum. In this paper, mathematical modeling of Inverted Pendulum-Cart system is designed and the step response of linearized model is analyzed in Matlab-Simulink platform. This justifies that the system is unstable. In order to stabilize the Inverted pendulum system, two linear controllers, pole placement and Linear Quadratic Regulator(LQR) are designed. These controllers are applied to both linearized and non-linear model. Comparative performance analysis reflects that linear controllers gives satisfactory performances for non-linear system yielding stability and robustness.

Keywords: *Inverted Pendulum, Pole Placement, Linear Quadratic Regulator, Linearized and Non-linear controller*

1. INTRODUCTION

The inverted pendulum is an unstable system with highlynon-linear dynamics. This system belongs to the class of U nder-actuated mechanical systems having fewer control inputs than degrees of freedom. This renders the control task of inverted pendulum as a challenge forming a classical benchmark for the designing, testing, evaluating and comparison of different classical and contemporary control techniques. Control of inverted pendulum has been a research interest in the field of control engineering as it offers wide range of applications. The numerous applications of inverted pendulum is deeply analyzed in [1] [2] where, the key motivations for the use of inverted pendulum system and throws main reflections on overall picture of historical, current and challenging developments based on the stabilization principle of the inverted pendulum. This paper also highlights vision-based human tracking control which paves the way for most efficient application in recent technology. Due to its importance in real world, this is a choice of dynamic system to analyze its dynamic model and to propose a suitable control law. The dynamic modeling of the system is well explained in different papers [3]. In this paper, modeling and control of non-linear coupled spacecraft-fuel system is discussed. The nonlinear interaction of the rigid body dynamics of a spacecraft with the sloshing dynamics of the liquid propellant in its partially filled spherical tank during an orbital transfer where fuel slosh dynamics are described by a three-dimensional two-pendulum model. Inverted pendulum with certain uncertainties can also be taken into consideration. The primary aim is to stabilize the inverted pendulum such that the position of the cart on the track is controlled accurately so that the pendulum is always erected in its inverted position during movements. Realistically, this simple mechanical system is representative of a class of altitude control problems such as rocket launching whose primary goal is to maintain the desired velocity oriented position at all times.

In general, all the control problems consists of obtaining suitable dynamic models of systems and using these models to determine control laws or strategies to achieve the desired system response and performance. The simplicity of control algorithm as well as to guarantee stability and robustness in the closed loop system is challenging task in real situations. Different controllers are being proposed in order to stabilize pendulum in upright position as in [4]. This paper highlights pole placement PI-state feedback controller design to control an integer order system. This is a simple method while the gain values of proportional and derivatives are high. A comparative study of different controllers based on stability and robustness is explained. in [5] [6]. This paper considers stabilisation and control of inverted pendulum on cart moving on an inclined surface. The task of controlling inverted pendulum on inclined surface is much complex and time consuming as compared to that moving on horizontal surface. An offline control of the proposed system is being been developed using proportional integral- derivative (PID) and fuzzy controllers. It is difficult to find the membership

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function used in fuzzy controllers which is a major disadvantage. Tracking control of inverted pendulum where the cart is shifted to final position and inverted pendulum position in upright is discussed in [7] [8] [9]. This paper also employs the importance of fuzzy neural networks which throws reflections on artificial intelligence which is the recent challenging technology. Another controller employed is sliding mode control where the chattering phenomenon strikes to its disadvantage as in [10]. All the literatures highlights about different linear and non-linear controllers. In this paper, mathematical modeling of inverted pendulum-cart system is designed and two linear controllers that is, pole placement and linear quadratic regulator (LQR) controllers are implemented. The controllers are applied to linear and non-linear models to analyze the performances. It is of paramount importance to analyse the stability and robustness of linear controllers to be applied to a non-linear system. This is achieved in this paper.

This paper is organized into six sections. First section accounts into introduction part where a brief information about the importance of inverted pendulum and different control strategies adopted are explained. Mathematical modeling of inverted pendulum and the controller designed to stabilize pendulum is explained in subsequent sections. The controller designed is pole placement controller and Linear Quadratic Regulator (LQR) which is a simple controller and is applied to both linearized and non-linear model. The effect of disturbance to the system is also explained. Performance analysis of linearized controller is analysed and conclusions are drawn.

2. MATHEMATICAL MODELING OF INVERTED PENDULUM-CART SYSTEM

In this section, the general mathematical modeling of inverted pendulum using the fundamental force balance and torque balance equations are explained. The standard statespace representation of fundamental equation is derived. Further, the system is linearized using Jacobian linearization technique and linear form of system equations is given in subsequent section.

A. Inverted Pendulum System Equations

Balancing the Inverted Pendulum-Cart system is in correlation with rocket launching, missile guidance and spacecraft systems where, the centre of gravity is incorporated behind the centre of drag leading to aerodynamic instability. This system consists of a cart which is motor driven to move along xaxis horizontally. The pendulum rod is fixed to the pivot point and the bob which is the other end is free to move along the vertical x-y plane. When the pendulum is forced to stay at upright position, it is called Inverted pendulum or IP system. This system is inherently unstable and requires continuous balancing force which can be denoted as F, supplied by the motor in practical situations or a proper controller to keep the pendulum upright. The free body diagram representation of an inverted pendulum mounted on a motor driven cart is shown in Fig.1.

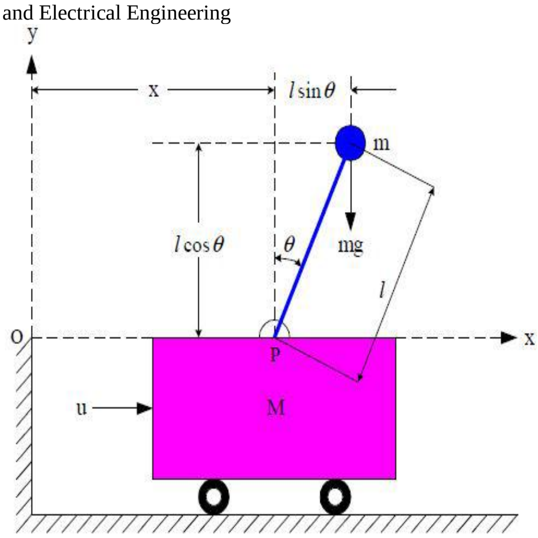

Fig. 1. Inverted Pendulum-Cart System [2]

Some assumptions are made such that the pendulum rod is mass-less and the hinge point is friction-less. The mass of the cart and the point mass of the pendulum bob at the upper end of the inverted pendulum are denoted as M and m, respectively. An external x-directed force on the cart is applied, u(t) and a gravity force always acts on the point mass. The coordinate system considered is shown in Fig.1, where *x(t)* represents the position of the cart and *θ(t)* is the tilt angle with respect to the vertically upward direction.

The fundamental force balance in the x-direction implies that the summation of force along the cart and point mass should be equal to the external force acting on the system. Force is given by the product of mass and acceleration. The time-dependent center of gravity of the point mass is given by (x_g, y_g).
where,

$$x_g = x + l\sin\theta$$
$$y_g = l\cos\theta$$

l=pendulum rod length

Force balance is thus given by,

$$M\frac{d^2x}{dt^2} + m\frac{d^2x_g}{dt^2} = u. \qquad (1)$$

On substitution of center of gravity into force balance equation (1) gives,

$$(M+m)\ddot{x} - mlsin\theta(\dot{\theta})^2 + mlcos\theta\ddot{\theta} = u. \qquad (2)$$

Similarly, the fundamental torque balance on the system is taken into consideration. Torque is given by the product of the perpendicular component of the force and the distance to the pivot point (lever arm length, *l*). The torque acting on the mass due to the acceleration force is balanced by the torque acting on the mass due to the gravity force. The resultant torque balance can be written as,

$$(F_x cos\theta)l - (F_y sin\theta)l = (mg sin\theta)l. \qquad (3)$$

The force components are determined as,

$$F_x = m\frac{d^2x_g}{dt^2} = m[\ddot{x} - lsin\theta\dot{\theta}^2 + lcos\theta\ddot{\theta}],$$

$$F_y = m\frac{d^2y_g}{dt^2} = -m[lcos\theta\dot{\theta}^2 + lsin\theta\ddot{\theta}].$$

On substitution of force components into equation (3) gives,

$$m\ddot{x}cos\theta + ml\ddot{\theta} = mgsin\theta. \qquad (4)$$

Equations (2) and (6) are the fundamental equations governing the inverted pendulum-cart system. It represents a nonlinear, complex, multivariable system which is relatively complicated from a mathematical viewpoint. However, since the goal of this particular system is to keep the inverted pendulum at upright position around θ= 0 with the application of cart force, the linearization must be considered about this upright equilibrium point. Force is applied on the cart and the cart moves in desired position. Linearization might be considered about this upright equilibrium point in order to stabilize the system.

The standard state space form is given by,

$$\frac{dx}{dt} = f(x, u, t).$$

To put defining equations into this form,
From equation (4),

$$ml\ddot{\theta} = mgsin\theta - m\ddot{x}cos\theta.$$

On substitution of equation (4) into equation (2) gives,

$$(M + m - mcos^2\theta)\ddot{x} = u + mlsin\theta\dot{\theta}^2 - mgcos\theta sin\theta, \quad (5)$$

$$(mlcos^2\theta - (M+m)l)\ddot{\theta} = ucos\theta - (M+m)gsin\theta + mlcos\theta sin\theta\dot{\theta}^2. \quad (6)$$

Dividing by the leading coefficients of equations (5) and (6) gives,

$$\ddot{x} = \frac{u + ml(sin\theta)\dot{\theta}^2 - mgcos\theta sin\theta}{M + m - mcos^2\theta}, \qquad (7)$$

and,

$$\ddot{\theta} = \frac{ucos\theta - (M+m)gsin\theta + ml(cos\theta sin\theta)\dot{\theta}^2}{mlcos^2\theta - (M+m)l}. \qquad (8)$$

Consider the state variables for representation in state space form as follows:

$$x_1 = \theta, \quad x_2 = \dot{\theta} = \dot{x_1}, \quad x_3 = x\ , \quad x_4 = \dot{x} = \dot{x_3}.$$

Final state space equation is given by,

$$\frac{dx}{dt} = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}. \qquad (9)$$

Where,

$$f_1 = x_2,\ f_3 = x_4,$$

$$f_2 = \frac{ucosx_1 - (M+m)gsinx_1 + ml(cosx_1 sinx_1)x_2^2}{mlcos^2x_1 - (M+m)l},$$

$$f_4 = \frac{u + ml(sinx_1)x_2^2 - mgcosx_1 sinx_1}{M + m - mcos^2x_1}.$$

The pendulum angle θand the cart position x are the variables of interest which should be controlled, then the output equation may be defined as,

$$y = Cx,$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}. \qquad (10)$$

Equations (9) and (10) represents complete state space representations of non-linear inverted pendulum-cart system.

B. Linearizing the Inverted Pendulum System

The linearized form of inverted pendulum using Jacobian matrix is given in this section.
Linearized form for the system is given by,

$$\frac{d}{dt}\delta x = J_x(x_0, u_0)\delta x + J_u(x_0, u_0)\delta u. \qquad (11)$$

Reference state for the system is defined with pendulum stationary and upright with no input force, that is, $x_0 = 0, y_0 = 0$. ∂x is the states, ∂u is the input cart force, $J_X(x_0, y_0)$and $J_u(x_0, y_0)$are the jacobian matrices for states and input. The components of jacobian matrices are determined systemically.
The elements constituting first, second, third and fourth columns of $J_X(x_0, y_0)$ are given by$\frac{\partial f1}{\partial x1}, \frac{\partial f1}{\partial x2}, \frac{\partial f1}{\partial x3}, \frac{\partial f1}{\partial x4}$ at initial conditions $(x_0\ _, u_0)$.
On combination of separate terms gives,

$$J_x(x_0, u_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-mg}{M} & 0 & 0 & 0 \end{bmatrix}. \qquad (12)$$

The derivative of non linear terms with respect to u is given by,

$$J_u(x_0, u_0) = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \\ \frac{\partial f_4}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}. \qquad (13)$$

After all these manipulations, linearized equation can be written as,

$$\frac{d}{dt}\delta x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-mg}{M} & 0 & 0 & 0 \end{bmatrix}\delta x + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix}\delta u. \qquad (14)$$

This is the open loop linearized model of Inverted pendulum with cart force $\partial u(t)$

3. LINEAR CONTROLLERS DESIGN

Two controllers are designed for inverted-pendulum system which is applied for both linear and non-linear model.

A.Pole Placement

Pole placement method is a classical controller design method to determine the places of the closed loop system poles on the complex plane. It is done by setting a controller gain K. Placing poles is desirable because the location of the poles corresponds directly to the eigen values of the system, which control the characteristics of the response of the system. The system must be considered controllable in order to implement this method. The desired locations of the poles are selected.

Consider a linear dynamic system in state-space form,

$$\dot{x} = Ax + Bu, \quad (15)$$
$$y = Cx.$$

It is able to achieve the goal (stabilizing or improving the transient response of the system) by using full state feedback, which represents a linear combination of state variables that is given by,

$$u = -Kx. \quad (16)$$

so that the closed loop system is given by,

$$\dot{x} = (A - BK)x, \quad (17)$$
$$y = Cx.$$

B.Linear Quadratic Regulator(LQR)

It is an optimal control technique concerned with the operation of a dynamical system at minimum cost. The main objective is to determine control signals that will cause the process to satisfy physical constraints to extremize performance index. This is a simple and robust strategy.Consider a linear dynamic system in state-space form,

$$\dot{x} = Ax + Bu, \quad (18)$$
$$y = Cx.$$

The state-feedback control is given by,

$$u = -Kx. \quad (19)$$

The closed loop system is given by,

$$\dot{x} = (A - BK)x, \quad (20)$$
$$y = Cx.$$

K is primarily derived from minimization of cost function,

$$J = \int (X^T QX + u^T Ru)dt. \quad (21)$$

where, Q and R are positive semi-definite and positive definite symmetric constant matrices respectively.

The LQR gain vector K is given by,

$$K = R^{-1}B^T P[3]. \quad (22)$$

where, P is a positive definite symmetric constant matrix which is obtained from the solution of Matrix Algebraic Reccatti Equation [3],

$$A^T P + PA - PBR^{-1}B^T P + Q = 0.$$

TABLE I
NUMERICAL VALUE OF PARAMETERS

Model parameters	Numerical value
Mass of the cart(M)	2.4 kg
Mass of the pendulum(m)	0.23 kg
Length of the pendulum(l)	0.36 m
Length of cart track(L)	+/-0.5 m
Friction coefficient of cart and pole rotation	Negligible

4. SIMULATION RESULTS

The Matlab-Simulink models for the modelling, stability analysis and for the controller design of nonlinear invertedpendulum-cart dynamical system have been developed. The parameters used for modeling inverted pendulum is shown in TABLE I. The frictional coefficient of cart and pole rotation is considered to be negligible. The fundamental equations governing inverted pendulum-cart system is given by,

$$\ddot{x} = \frac{u + ml(sin\theta)\dot{\theta}^2 - mgcos\theta sin\theta}{M + m - mcos^2\theta},$$

and,

$$\ddot{\theta} = \frac{ucos\theta - (M+m)gsin\theta + ml(cos\theta sin\theta)\dot{\theta}^2}{mlcos^2\theta - (M+m)l}.$$

On substitution of different modeling parameters to the fundamental equations gives,

$$\ddot{x} = \frac{u + (0.23 * 0.36)(sin\theta)\dot{\theta}^2 - (0.23 * 9.8)cos\theta sin\theta}{(2.4 + 0.23) - 0.23 * cos^2\theta},$$

$$\ddot{\theta} = \frac{ucos\theta - (2.4 + 0.23)9.8sin\theta + (.23 * 0.36)(cos\theta sin\theta)\dot{\theta}^2}{(0.23 * 0.36)cos^2\theta - (2.4 + 0.23)0.36}.$$

As the system is non-linear, linearization is considered about the equilibrium point. After linearization,the system matrices used to design are computed as below:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 29.8615 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.9401 & 0 & 0 & 0 \end{bmatrix}, \quad (23)$$

$$B = \begin{bmatrix} 0 \\ -1.1574 \\ 0 \\ 0.4167 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The non-linear model of inverted pendulum-cart system is shown in Fig.2 where, a step input is applied as input. A disturbance input is also applied at 4 secs. The step response of linear and non-linear model is also taken into account. The step response of linearized model is shown in Fig.3and Fig.4. Fig.3 gives the output responses of pendulum angle and cart position. Starting from initial positions, they approaches infinity. Also, the different states of pendulum deviate in an indifferent manner. It infers that the system is inherently unstable. After analyzing the stability of system, two controllers are designed that is, pole placement and LQR. These controllers are applied to the linearized as well as nonlinear model of inverted pendulum for stability analysis.

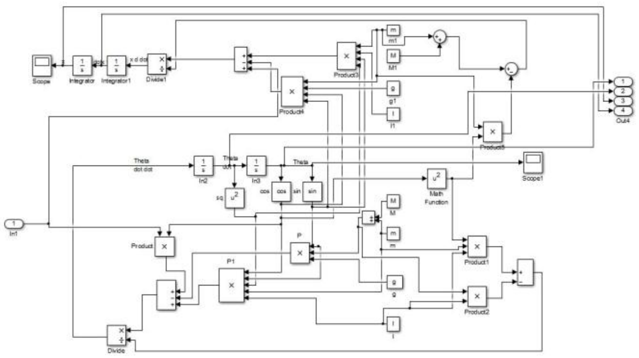

Fig. 2. Non-linear model of inverted pendulum system

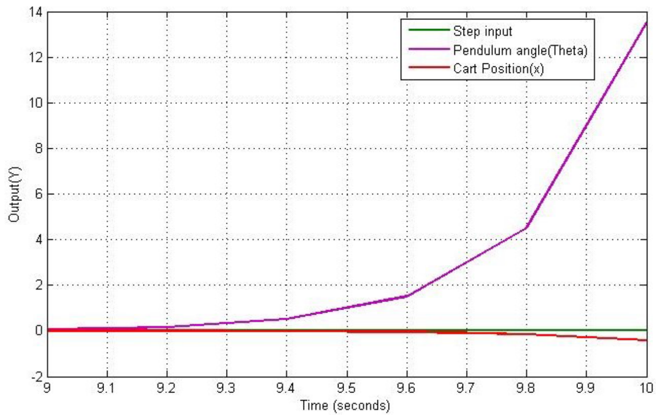

Fig. 3. Output response of inverted pendulum system

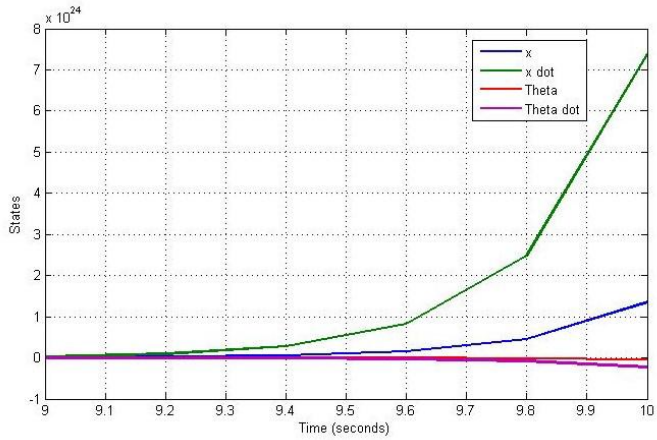

Fig. 4. States of inverted pendulum system

A. To Linearized Model of inverted pendulum

The application of pole placement and LQR controllers to the jacobian-linearized model is given in this section.

1.Pole Placement: The linear model of inverted pendulum system with the application of pole-placement controller is developed. The desired pole locations are at [-5 -7 -2+j2 -2-j2]. These values in the left half plane is chosen at random. The controller value is obtained as [-113.30 -21.30 -24.65 -20.78]. The output responses are pendulum angle θ and cart position (x). The initial value of pendulum angle is taken as 5 radians. The disturbance input is applied at 4 sec. The responses of inverted pendulum system with pole placement technique is shown in Fig.5. Initially both the pendulum angle and cart position is subjected to oscillations and is stabilized to zero at 3 secs. As the disturbance is applied at 4 secs, it again shows oscillations for about 2 secs and stabilizes to zero. Total settling time accounts to 6 secs after the application of disturbances.

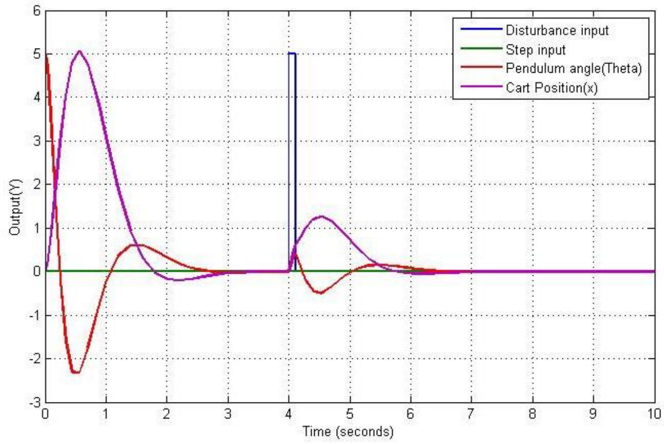

Fig. 5. Output responses of system with disturbance for pole placement controller

2).Linear Quadratic Regulator: The choice of Q and R is computed as follows [4] :

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 250 \end{bmatrix}, \quad (24)$$

$$R = 1.$$

LQR gain vector is obtained using equation [22] as follows:

$$K = \begin{bmatrix} -137.7896 & -25.9783 & -22.3607 & -27.5768 \end{bmatrix}. \quad (25)$$

The linear model of inverted-pendulum system with the application of linear quadratic regulator controller is developed. The output responses are pendulum angle (θ) and cart position (x). The initial value of pendulum angle is taken as 5 radians. The disturbance input is applied at 4 secs. The responses of inverted pendulum system with LQR technique is shown in Fig.6. Initially both the pendulum angle and cart position is subjected to oscillations and is stabilized to zero at 3.8 secs. As the disturbance is applied at 4 secs, it again shows oscillations for about 2 secs and stabilizes to zero. Total settling time accounts to 7 secs after the application of disturbances.

B. To Non-Linear Model of inverted pendulum

The application of pole placement and LQR controller to the non-linear model of inverted pendulum is performed.

Pole Placement and Linear Quadratic Regulator Controller: Pole placement and LQR controller is implemented with the gain values similar to that applied for a linear model. That is, the gain value for pole placement controller is taken as [-113.30 -21.30 -24.65 -20.78] and LQR gain vector is taken as K= [-137.7896 -25.9783 -22.3607 -27.5768]. Disturbance input is applied at 4 secs. The output responses of controller with the application of disturbance input is shown in Fig.7 and Fig.8. Initially, the pendulum angle and cart position oscillates for small interval of time and after the application of disturbance input, it stabilizes to zero at 5 secs. There is very low peak overshoots with the application of both controllers. The inference is that the non-linear model also gives satisfactory performances with the application of linear controllers. The possible reason can be that this system can be described pretty accurate by linear approximation at a specific operating point. In additon, the actual non-linear system may approaches the linearized system.

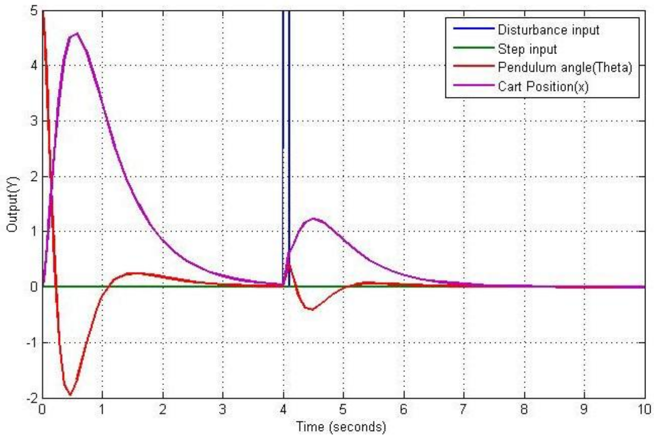

Fig. 6. Output responses of system with disturbance input for LQR controller

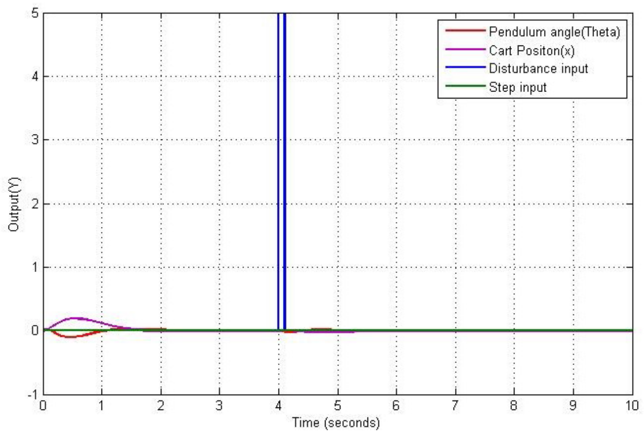

Fig. 7. Output responses of system with disturbance input for pole placement controller

TABLE II
PERFORMANCE ANALYSIS

LINEAR MODEL		
Parameters	Pole Placement	LQR
Settling time Peak overshoot(cart) Peak Overshoot(Pendulum)	6 sec 5 m -2.5 rad	7 sec 4m -1.8 rad
NON LINEAR MODEL		
Settling time Peak overshoot(cart) Peak Overshoot(Pendulum)	5 sec 0.18 m -0.12rad	5 sec 0.17m -0.1rad

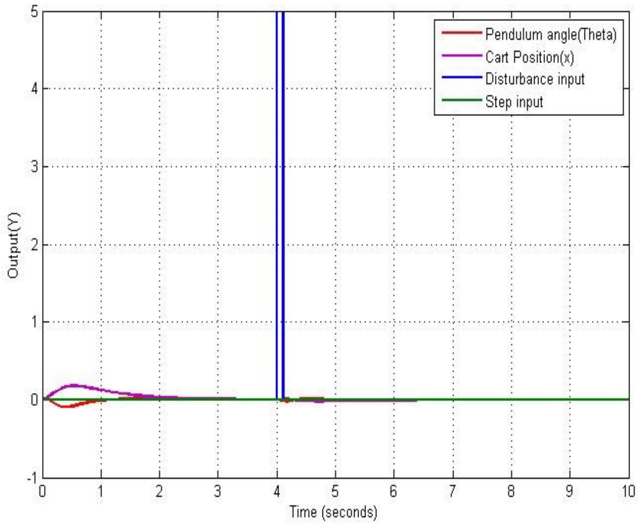

Fig. 8. Output responses of system with disturbance input for LQR controller

5. COMPARISON OF PERFORMANCE

The performance parameters such as settling time and peak overshoot is compared in TABLE II. It is observed that the application of linear controllers to non-linear model givessatisfactory performances. The non-linearities which existed in actual model vanishes and becomes stabilized after a short interval of time. The possible reasons can be the linearization technique adopted (Jacobian linearization), which is done for the equilibrium point (0,0). The selection of operating points is one such reason for this stabilizing behaviour. The qualities which determines the best performance of a system such as stability and robustness is achieved.

6. CONCLUSIONS AND FUTURE SCOPE

Mathematically modeled the inverted pendulum-cart system which offers global applications and further two linear controllers are designed. The output responses of pole placement controller and Linear Quadratic Regulator(LQR) controller which is applied to both linearized and non-linear model infer satisfactory performances. Settling time and peak overshoot with both controllers are comparatively same for linear model. Non-linear model is also stabilized with linear controllers. The possible reason can be the linearization done about one equilibrium point. More studies can be done on nonlinear controllers and learning based controllers to analyze the stability and robustness of the system. In addition to that, practical implementation of this system should be taken into account for comparing the numerical and experimental results. Also, to choose the best controller in real-life applications.

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